

Stability of dynamical systems with discontinuities

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The dimensional model of many parametric dynamic system with discontinuity are considered. The stability boundary are calculated analytically and numerically. New typical features of stable cycle structures provoked by discontinuity are found.

Рассмотрена одномерная модель многопараметрической динамической системы с разрывами. Вычислены аналитически и численно границы зон устойчивости. Обнаружены типичные особенности в формировании структуры устойчивых циклов, связанные с наличием разрыва.

The stability of physical systems is a very important question for many applications. The regular and the chaotic motion as well as a transition between them are often studied using the point mapping method [1]. Area of stable or (quasi) periodical motion of this mapping get long-live and hence operational state for physical systems. It is obvious, that the determination of boundaries and type of stability are also very important. In this paper for 1D-model dynamical system (piecewise linear mapping) we consider the typical structure of stability area and features caused by discontinuity mapping, representing the model of sudden and jump-like processes in a physical system.

Let us consider a quasi linear, discontinuous mapping on the interval $I = [0,1]$

$$x_{n+1} = f(x_n);\ \ f(x) = \sum_{i=1}^{4}(\mu_i x + \lambda_i)\theta(x_i - x)\theta(x - x_{i-1}), \quad (1)$$

with point separation $x_0 = 0$, $x_1 = a$; $x_2 = 1/2$; $x_3 = 1 - a$; $x_4 = 1$ and parameters $\mu_1 = \mu_4 = A/a$; $\mu_2 = \mu_3 = 2(E - A)/(1 - 2a)$; $\lambda_1 = 0$; $\lambda_2 = (A - 2aE)/(1 - 2a)$; $\lambda_3 = 1 + (A - (2(1 - a)E)/(1 - 2a)$ and $\lambda_4 = 1 - A/a = 1 - \lambda_1$. Coordinates of critical point (maximum) $(a, A = f(a))$ and of the break $E = f(1/2 - 0) = (1 + \Delta)/2$ with a gap $0 \le \Delta \le 1$ are independent parameters. To simplify the problem we can take a symmetrical mapping $f(x) = 1 - f\ (1 - x)$. We examine the dynamics with $0 < a < 1/2 \le A \le 1$. The non-monotony together with discontinuity as well as many parametrical feature of this mapping (1) differ it from another 1D-models [2].

The area of stable motion corresponds to stable periodical trajectory (cycles) of the mapping (1). The cycle is determined by a set of points $C = \{x_k\}_0^{p-1}$, so as $f^m(x_k) \ne x_k$, $m = 1, \ldots, p - 1$; $f^p(x_k) = x_k$ (p — period, x_0 — initial point); $f^p = f \circ \ldots \circ f - p$ — multiple mapping convolution. The cycle stability is determined by its multiplicator