Stochastic dynamic models of lattice gases

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It is developed an approach to the study of Markov processes with local interaction which model dynamics in so-called lattice gases. Such an approach to the study of non-equilibrium processes by means of statistical methods has some preferences connected with a simplicity of corresponding mathematical constructions. The simplest example of those systems has been analyzed named the ideal lattice gas. It is found it equilibrium probability distribution that it is the random lattice field with independent values which are distributed according to Poisson's distribution with equal to a lattice gas density.

В работе строится подход к изучению марковских процессов с локальным взаимодействием, которые моделируют динамику в системах, называемых решёточными газами. Такой подход к исследованию неравновесных процессов методами статистической физики обладает рядом преимуществ, связанных с относительной простотой соответствующих математических конструкций. В качестве простейшего примера разобран случай стохастического идеального газа на решётке. Вычислено его равновесное распределение вероятностей, которое является случайным полем с независимыми значениями, распределёнными по Пуассону с показателем, равным плотности решёточного газа.

Introduction

When solving the fundamental problems of the non-equilibrium statistical mechanics, it is just the Liouville evolution equation for the distribution density in the phase space of the corresponding determinated Hamiltonian system with each motion having the property of reversibility that is taken as the basis starting from [1]. According to the determinacy character of the initial system, the Liouville equation describes a prognosticable random process. In this case, the investigation purpose is to reveal the evolution irreversibility property of a large thermodynamic system and to derive equations for quantities providing a complete description of its final stage. In this path, in particular, it is to substantiate the trend to statistical equilibrium, that is, the trend towards the corresponding Gibbs distribution.

It is the Glauber model [2] that is the first dynamic model in statistical physics that, in contrast to the above-mentioned approach, is based ab initio on probabilistic considerations and therefore exhibits an evolutional irreversibility. The introduction of that model into statistical physics was caused by that its Hamiltonian was of a "too model" character, i.e. it was not determined on a manifold; thus, it was impossible to use in a natural manner the Poisson bracket in the phase space of the system and to construct the Hamiltonian dynamics basing thereon. Thus, the dynamics in the Glauber system was to be constructed by necessity basing on methods of the random process theory.

Later, a number of similar models was developed in statistical physics. The evolution therein is a random process, a Markovian one as a rule, in general in an infinite-dimesional state space. This is a significant distinction from finite-dimesional Markovian processes that are used traditionally in statistic physical problems, e.g., in the Wiener process or in Ornstein-Uhlen-