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A theoretical model for calculation of biphase composite failure energy

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A conjunction of two approaches to the estimation of composite material strength characteristics is reported. The first approach is based on taking into account the influence of crack front bending between two stoppers (delay sites). In this case, the strength is increased because of the additional energy expenditure for the crack front elongation. The second approach is based on the evaluation of the influence of physical and mechanical characteristics of both phases as well as upon the character of the stopper overcoming. The presented model has been used to calculate the failure energy of Si_3N_4-SiC composite with different grain size ratios.

В работе представлено объединение двух подходов к оценке прочностных характеристик композиционных материалов. Первый подход основан на учёте выгибания фронта трещины между двумя местами задержки (стопорами). При этом прочность повышается в результате добавочных энергетических затрат на удлинение фронта. Сущность второго подхода заключается в учёте влияния механических характеристик обеих фаз на характер преодоления трещиной стопоров. Представленная модель использована для оценки энергии разрушения композита Si_3N_4-SiC с разными соотношениями размеров зерен обеих фаз.

The majority of composite fracture toughness and failure energy calculations [1-3] is based on consideration of a crack with direct front which progressively crosses different grains. On the other hand, F. Lange [4] analyzing a microphoto of a MgO single crystal failure surface, judges that a crack front becomes curved between the delay places before the final failure. The size of that bend has offered the basis for a model according to which the "overcoming" of delay sites by a crack occurs when the crack front between two delay sites (stoppers) reaches the semicircular shape. In [5], an attempt has been made to estimate mechanical characteristics of the stoppers. In this work, an attempt is made to associate the approaches taken from [4] and [5] and the model is proposed for calculation of failure energy for a biphase composite where the crack can be curved between the delay sites which have different sizes and certain mechanical characteristics.

Let us consider a crack in a material with structure parameter D under loading (Fig. 1). Integrating the expression for elastic energy density ($W = \sigma^2/2E$) in the vicinity D of a crack front, we obtain an expression for elastic strain energy:

$$TL = \frac{\alpha L D K_1^2}{E} \qquad (1)$$

where L is the crack front length; T, the elastic straining energy of the material per unit front length; E, Young modulus for the material; $\alpha \approx 9/16$.

The material failure before the crack front will take place when the stress inten-

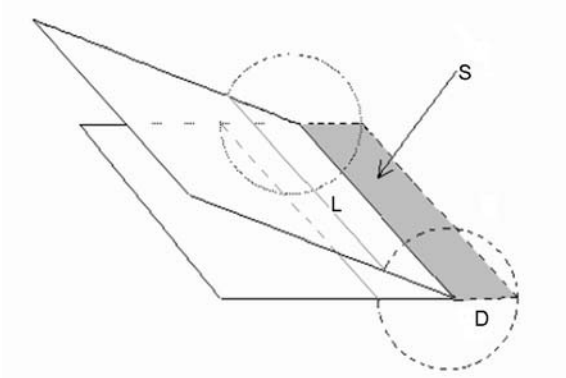

Fig. 1. A crack tip.

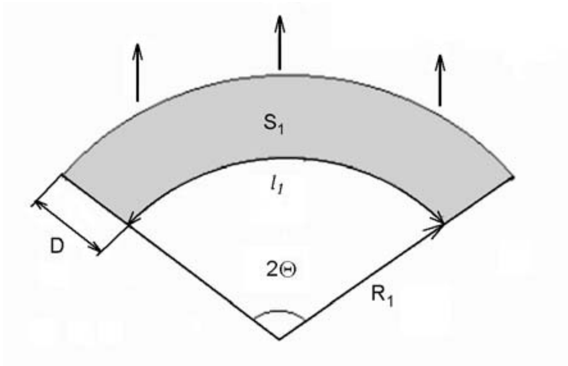

Fig. 2. Form of a crack front in matrix.

sity coefficient reaches the critical size $K_1 = K_{1C}$. Thus,

$$T_c = \alpha D\gamma_0,$$

where $\gamma_0 = K_1^2/E$ is the failure energy per unit surface area of the material. In other words, for failure, the accumulation of elastic energy is necessary "on the area" $S = LD$ before the front,

$$T_c L = \alpha S\gamma_0. \quad (2)$$

Let obstacles be exist before the crack front, the sizes thereof being $2d$, the average spacing of the obstacles being L. According to [4], the front line bends between the obstacles (the front length f will increase up to L_1) before the failure begins. It is clear that the bending will take place when the stress intensity coefficient becomes sufficient for the matrix phase failure but insufficient to overcome the obstacles. During the crack front bending, energy is spent not only for formation of the failure surface, but also on the front elongation, and thus, linear elastic energy T_1 increasing, then:

$$\gamma_1^* dS = \gamma_1 dS + T_1 dl_2, \text{ or} \quad (3)$$
$$\gamma_1^* = \gamma_1 + T_1 dl_2/dS$$

where γ_1 is the matrix phase failure energy without taking the bending into account; γ_1^*, the effective failure energy of the matrix phase in view of additional energy for the bending dissipation. The crack front shape between stoppers is shown in Fig. 2. For the crack front propagation between the delay sites, it is necessary that the elastic energy value on a site in width D before the front had become:

$$T_1 l_1 = \alpha S_1 \gamma_1^*. \quad (4)$$

Then the effective value of matrix failure energy from (3) and (4) is:

$$\gamma_1^* = \gamma_1 + \alpha S_1\gamma_1^*/l_1R_1 \Rightarrow \gamma_1^* = \quad (5)$$
$$= \gamma_1(1 - \alpha S_1/l_1R_1)^{-1} \approx \gamma_1\exp(\alpha S_1/l_1R_1).$$

Let us consider that the delay site has spherical shape of radius d and mechanical characteristics E_2, γ_2, K_{1C2}. For failure of a stopper, the elastic energy should be accumulated on S_2 (Fig. 3) :

$$T_2 l_2 = \alpha S_2\gamma_2. \quad (6)$$

The condition of the crack propagation on the whole front is the simultaneous fulfillment of conditions (4) and (6), that is,

$$\frac{T_1l_1}{T_2l_2} = \frac{S_1l_1}{S_2l_2}\exp(\alpha S_1/l_1R_1) \quad (7)$$

Let us replace the real crack front by a front which shaped as a broken line (Fig. 4). Let the linear energy on sites L_1 and L_2 be designated as T_1^* and T_2^*. The relation between T_1^* and T_2^* depends on parameter h which let be selected so that L_1h_1 and L_2h_2 are equal to areas of the respective segments. We shall put, according to (5):

$$T_1l_1 \approx Tl_1(1 - exp(-\beta L_1/2h)), \quad (8)$$
$$T_2l_2 \approx T(l_2 + l_1exp(-\beta L_1/2h))$$

where T_1 and T_2 are linear energies on the curved sites l_1 and l_2, respectively; $\beta = E_1/E_2$.

Having accepted the above assumptions, we shall write down a condition (7) as:

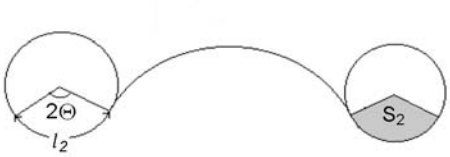

Fig. 3. A crack front between two stoppers

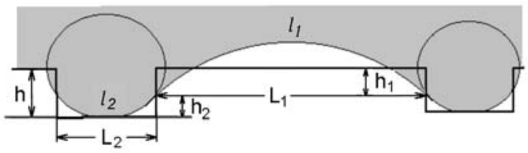

Fig. 4. Replacing of a real front with broken line.

$$\frac{L_1/L_2 + \exp(-\beta L_1/2h)}{1 - \exp(-\beta L_1/2h)} = \frac{S_2\gamma_2}{S_1\gamma_1}\cdot exp(-\alpha S_1/L_1R_1)$$

Or:

$$\frac{l_1/l_2 + \exp(-\beta L_1/2h)}{1 - \exp(-\beta L_1/2h)} \quad (9)$$

$$\frac{S_1\gamma_1}{S_2\gamma_2}\cdot exp(-\alpha S_1/L_1R_1) = 1.$$

Here,

$$R_1 = (L - d\cdot\sin\Theta)/2\sin\Theta,\ S_1 = \Theta D(2R_1 + D).$$

The expression (9) is in fact a condition of the beginning of catastrophic propagation of a crack along the whole front. On a site l_2, area S_2 (see Fig. 4) should be calculated as formula $S_2 = \Theta d^2$. But it is obvious that, when we have a rectilinear front (at the beginning of bending), S_2 has the rectangular shape at width D. Considering that in the course of bending, the shape of S_2 is approached a sector rather fast, but without jumping, we shall write:

Having calculated Θ from (9), it is possible to estimate the effective failure energy of a composite:

$$S_2 = 2\Theta d(D + \frac{1}{2}(d - 2D)(2\Theta/\pi)^{1/16}$$

$$\gamma_{eff} = \frac{L_1}{L}\gamma_1\exp(\alpha S_1/l_1R_1) + \frac{L_2}{L}\gamma_1. \quad (10)$$

Here, it is necessary to consider a situation when $\gamma_{20} < \gamma_{10}$. As it follows from [5], in case of presence of inclusion having a higher Young modulus ($E_{20} > E_{10}$) and a lower failure energy ($\gamma_{20} < \gamma_{10}$) than those of the matrix, the stopper effect is realized at the matrix/inclusion boundary. In this case, at insignificant differences between E_{10} and E_{20}

$$\gamma_2 \approx \gamma^*_{10}E_{20}/E_{10} \quad (11)$$

where, under the condition of strong contact between phases 1 and 2, γ^*_{10} is the failure energy of the pore-free matrix material.

Thus, inclusions of the second phase can serve as delay sites for propagation of a crack front (if it is stopped in the face of the inclusion at its boundary) and as areas of the material weakening (if the front is inside the inclusion). To simplify the taking into account these effects, it is possible to consider that the certain part (θ_1) of second phase grains has failure energy γ_2, and the other grains, energy γ_{20}. Thus, the matrix "phase" consists of particles with mechanical characteristics γ_{10}, E_1, and γ_{20}, E_{20}.

Let us consider a composite consisting of two phases with certain mechanical characteristics. There is a critical angle θ (from (9)) and the parameter of deflection h at which a stopper is broken. Let us construct a rectilinear front line in the material. The average distance between particles of phase 2 which will be crossed with this line will be $L = 2d/\theta$ (where d is the paerticle radius, θ, the content of the 2-nd phase particles). At $h > 2d$, all the particles which are crossed with a front line of a crack are stoppers. At $h < 2d$, those particles which fraction of total will make $\sim h/2d$ will be the delay places. Then the fraction of stoppers $\theta_1 \approx \theta h/2d$. In this case, an average distance between stoppers is $L = d/\theta_1 = 2d^2/\theta h$. Under the condition $h \approx L\Theta/6$, $L \approx d(12/\theta\Theta)^{1/2}$. Then the value γ_1 (from (10)) can be appreciated as:

$$\gamma_1 \approx \frac{L\gamma_{10} - d\gamma_{20} - (\gamma_{10} - \gamma_{20})\theta L}{L - d} \quad (12)$$

Similarly,

$$E_1 \approx \frac{LE_{10} - dE_{20} - (E_{10} - E_{20})\theta L}{L - d} \quad (13)$$

Thus, $E_2 = E_{20}$, $\gamma_2 = \gamma_{10}E_{20}/E_{10}$.

In [4], the failure energy of Si_3N_4-SiC composite is investigated depending on SiC contents at various ratios of the average grain sizes of both phases. On the plot (Fig. 5),

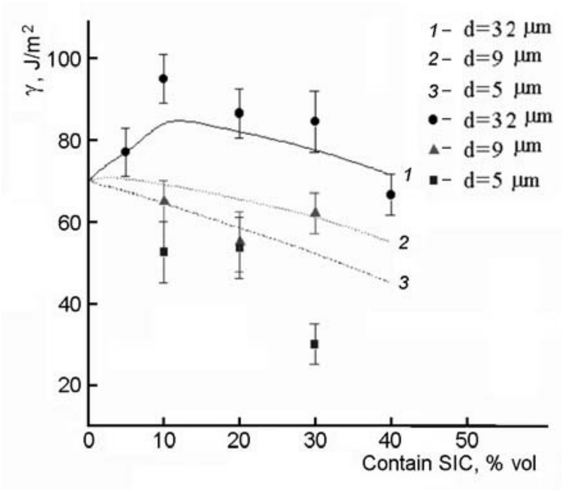

Fig. 5. Destruction energy of Si3N4-SiC composite. The theoretical dependences obtained with formula (9) - (13) and experimental points from [4].

the experimental points and the dependences obtained with formulae (9)–(13) are shown. In calculations, the following characteristic values for the materials are used: γ_{10} = 70 J/m^2 (failure energy of pure Si_3N_4 obtained by F. Lange [4] for material synthesized under conditions identical for all researched composites), γ^*_{10} = 100 J/m^2 (failure energy of pore-free Si_3N_4 [7]), γ_{20} = 20 J/m^2 (failure energy of SiC [7]), E_{10} (Si_3N_4) ≈ 320 GPa, E_{20}(SiC) ≈ 440 GPa. From the plots, it is seen that the proposed model displays the qualitative picture of experimental data rather well. Insignificant quantitative deviations for the composite with the inclusion size of $2d \approx 32$ μm can be connected with the choice of interrelation between T_1 and T_2 (8), and with a margin error in measurement of the phase 2 average grain size. The deviation of the curve for the inclusion size of $2d \approx 5$ μm can be connected with almost full absence of mutual connection between SiC grains under the conditions of synthesis (taking into account significant differences between synthesis temperatures of Si_3N_4 and SiC), the contact probability increasing sharply [8] at the identical sizes of Si_3N_4 and SiC grains. Moreover, the residual microscale stresses connected with distinction of thermal expansion coefficients of different phases are not taken into account in our work.

Thus, the model proposed makes it possible to estimate qualitatively and quantitatively the failure energy and fracture toughness of fragile multiphase ceramic materials. Consideration of the crack front bending has enabled to explain the fact of the material strengthening due to introduction of inclusions with lower value of failure energy, and also to explain dependence of the effect on a ratio of matrix/inclusion grain sizes.

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Теоретична модель розрахунку енергії руйнування двофазного композита.

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У роботі представлено об'єднання двох підходів до оцінки міцнісних характеристик композиційних матеріалів. Перший підхід засновано на урахуванні вигинання фронту тріщини між двома місцями затримки (стопорами). При цьому міцність підвищується у результаті додаткових енергетичних витрат на видовження фронту. Сутність другого підходу полягає у врахуванні впливу механічних характеристик обох фаз на характер подолання тріщиною стопорів. Представлена модель використана для оцінки енергії руйнування композита Si_3N_4-SiC із різними співвідношеннями розмірів зерен обох фаз.