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Modeling of diffuse reflection via billiards

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Method of modeling of diffuse-mirror reflection of beams using microscopic billiards is offered. It is shown, that such modeling is possible using topologically unusual disseminating %billiards on the cylinder. Qualitative changes of dispersion indicatrix are discussed with variation of the parameters characterizing the form of a billiard.

В работе предложен метод моделирования диффузно-зеркального отражения лучей используя микроскопические биллиарды. Показано, что такое моделирование возможно при использовании топологически необычных рассеивающих биллиардов на цилиндре. Обсуждены качественные изменения индикатриссы рассеяния при варьировании параметров, характеризующих форму биллиарда.

1. Introduction

Mathematical billiards - one of classes of dynamical systems actively studied now. Elementary billiards already generate dynamics leading to occurrence of determined chaos [1], [2]. It is well-known, that any difficult enough system usually shows chaotic behavior. Therefore studying of laws of chaotic regimes is important. Mathematical billiards are one of the most simple systems on which it is convenient to study mechanisms of occurrence of chaos and features of chaotic behavior. In physics billiards are applied to modeling of properties of many real systems.

One of the most natural and known applications of billiards to real physical problems, is modeling by means of billiards of distribution of light in scintillation crystals [3], [4]. Distribution of light in a crystal corresponds to rectilinear movement of a beam in billiards, and to reflection of light from crystal border there corresponds reflection of a beam from billiards border. At the description of light harvesting in scintillation crystals the detector is identified with a corresponding billiard. Dynamics of light beams is reduced to research of billiard trajectories, and the associated with them topological and metric characteristics. The picture of light harvesting in general is defined by properties of a billiard. Studying of them allows clarifying the general physical laws of light harvesting and, as a result, more effectively optimize this process [5].

However, modeling of such processes face a problem connected with type of reflection of beams from border. In mathematical billiards reflection strictly follows the mirror law, while in real crystals light is reflected from crystal border diffusively. Real indicatrix of reflection has narrow maximum in a direction of mirror reflection and essential diffuse part [6]. Distribution of energy of falling light between mirror and diffuse reflection components depends on the angle of incidence, while the form of diffuse parts of indicatrix of dispersion, as a rule, is considered independent from the angle of incidence [6].

In this work it is shown, that light distribution in scintillation crystals even taking into account diffuse part of reflection can still be simulated via specific billiards in which reflection of a beam from borders occurs strictly under the mirror law.

2. Problem statement

Let's show how it is possible to model diffusive reflection component using billiards. For modeling of propagation of light in scintillation crystal we will use a billiard which as a whole has the form of crystal. However, its border is arranged so only on large scales. On small scales it consists of «microscopic» open billiards of some definite form (as on Fig. 1 for example) so that in general the billiard modeling a crystal will look like shown on a Fig. 2. The choice of the form of a microscopic billiard plays important role. In the chosen example the border of a microscopic billiard has disseminating site. Presence of such site in the closed billiard guarantees chaotic behavior of beams in it. The idea of modeling of diffusive component of dispersion is based on it.

The form of a microscopic billiard defines shape of indicatrix of dispersion. Even for the presented simple example the form of a microscopic billiard is defined by three dimensionless parameters. Having accepted as unit of length the distance between lateral walls l and normalize on it all distances, we will choose as billiard parameters height $h = H/l$, length of a horizontal part of the border adjoint to the wall $a = A/l$, and radius of curvature of a convex part of border $r = R/l$. Three dimensionless parameters create wide opportunities for control of form of dispersion indicatrix by variation of the form of microscopic billiards. Thus, for diffuse component of reflection to appear we will use as microscopic - billiards with chaotic behavior of the beams, in particular disseminating billiards [1]. Certainly, to provide access of beams to an interior of a corresponding billiard, we will consider the billiards belonging to a separate class - open billiards [8]. In this work we will consider only two most simple forms of microscopic billiards for demonstration of the basic idea of modeling of diffuse and mirror dispersion components. Let's start with microscopic billiard with border on the average convex, i.e. disseminating open billiard shown on Fig. 1. The size of «microscopic» billiard, and, accordingly, number of such billiards in unit length of a wall of global billiards, we will choose so that the size of microscopic billiard was small in comparison with width of the beam of light which distribution will be modeled, but much greater than the wavelength.

Let's notice, that the chosen form of billiard automatically provides the mirror component of reflection arising due to reflection from the top flat part of the border. The part of energy, corresponding to this

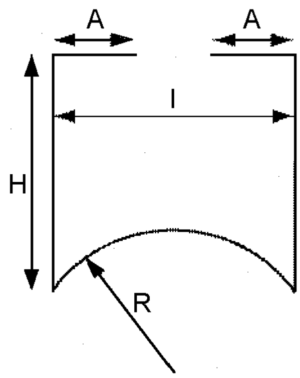

Fig .1. Example of form of «microscopic» billiard, forming large-scale border of a billiard. Such «microscopic» billiard is characterized by three dimensionless parameters $h = H/l$, $a = A/l$ and $r = R/l$. Here R - radius of curvature of a convex segment.

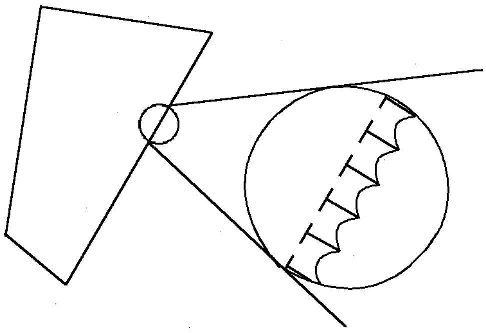

Fig. 2. General view of billiard and its border, which consists of «microscopic» billiards. In a circle the small-scale structure of border is shown.

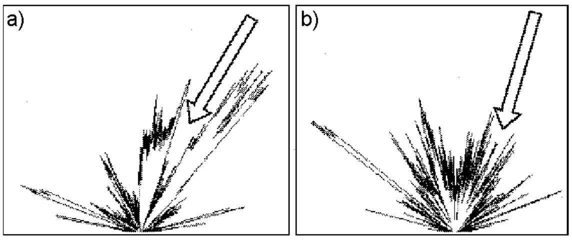

Fig. 3. Indicatrixes of dispersion for microscopic billiard which has lateral walls, with parameters a) $h = 0.66$, $r = 1$, $a = 0.3$ and b) $h = 0.66$, $r = 1$, $a = 0.46$, received as a result of numerical simulation. By the arrow the direction of an incident bunch of beams is shown. The length of indicatrix in the specified direction is proportional to number of the reflected beams in this direction.

part of reflection does not depend on the angle of incidence of beams. Besides, if we make the chosen microscopic billiards closed, having put the size of an entrance window equal to zero, such billiards obviously will be chaotic, due to presence of a convex disseminating part of border. One of properties of chaotic movement is «loss of memory» about initial conditions. Therefore for the beams which stay long enough in microscopic billiard, the probability to exit from it in a certain direction practically will not depend on position and angle of the beam, which it has entering billiard. Existence of trajectories with such property will provide diffuse reflection component. These simple reasons also define character of quantitative changes of diffuse component with change of the size of an entrance window and curvature radius r.

3. Properties of dispersion indicatrix

Using the border consisting from introduced above microscopic billiards (Fig. 1), it is possible to simulate reflection of beams from such border. Received as a result of numerical simulation typical indicatrixes of dispersion, generated by microscopic billiards, are shown in a Fig. 3. The characteristic form of indicatrix has maximum in a direction, close to the opposite direction of a falling bunch of beams. Collateral local maxima and diffuse part are also observed.

Distribution of energy between diffuse part and local maxima of reflection essentially depends on the form of microscopic billiard. In particular, if the size of an entrance window is small in comparison with characteristic size of billiard l, and the beam which has got into billiard suffers significant number of collisions before leaving it, than indicatrix of reflection will mainly be diffusive (see rice 3 b). It is possible to see comparing indicatrix of dispersion with narrower window and wider entrance window in Fig. 3 a. Respective alterations of dispersion components are well appreciable.

If the entrance window is wide enough, comparable with the characteristic size of billiard, than most part of falling beams will leave billiard after only several reflections from its walls. In this case the number

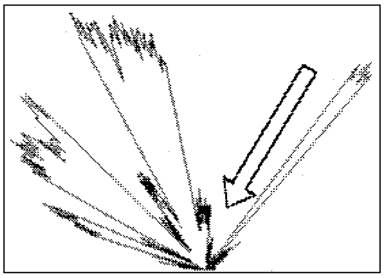

Fig. 4.With big enough entrance window reflection mainly occurs in several depending on angle of incidence directions. Indicatrix of dispersion for billiard with lateral walls and parameters $h = 0.66$, $r = 1$, $a = 0.13$ is shown.

of reflections will be insufficient for the mechanism of loss of memory of initial conditions to work, therefore the dispersion indicatrix will consist of several major directions of reflection (see Fig. 4). The number of such directions depends on the angle of incidence. For understanding of the mechanism of formation of these directions we will consider the beams leaving a microscopic billiard after fixed number of reflections. In this way the falling bunch of beams is divided into sub bunches each leaving billiard after some certain number of reflections from border of microscopic billiard. This sub bunches form corresponding splashes of the indicatrix of dispersion, each occupying some range of angles of reflection. With the increase of number of reflections before exit from the billiard, as the memory loss grows, the directions start to be distributed more and more homogeneously.

Following these representations we will consider the trajectories leaving billiard after one collision (Fig.5). It is easy to understand, that there is some critical angle φ_{cr}, such that if the bunch of beams falls under an angle $\varphi < \varphi_{cr}$ than trajectories leaving after one collision do not exist, and at $\frac{\pi}{2} > \varphi > \varphi_{cr}$ they are obviously present. For definition φ_{cr} we will notice, that if such bunch exist, it necessarily contains a trajectory reflected from the center of a convex part of border, i.e. from a point with co-ordinates $(\frac{1}{2}, r)$. The critical angle is defined by a trajectory touching right edge of an entrance window and passing through the central point of a disseminating segment (Fig. 5). It allows to calculate a critical angle $\varphi_{cr} = \arctan \frac{h+\sqrt{r^2-\frac{1}{4}}-r}{\frac{1}{2}-a}$ from simple geometrical reasons. Thus, the considered bunch always is reflected in the range of angles containing an angle of mirror reflection. Hence, the part of beams leaving billiards after one collision brings the contribution in mirror reflection component.

Let's estimate now, what part of energy of falling beams has this bunch. As a measure characterizing this energy, we will choose the attitude of width of a bunch reflected after one collision to width of entrance window. It is easy to see, that this share is equal $\frac{x}{1-2a}$, where x - dimensionless distance from a right edge of an entrance window to the beam which after reflection gets to a windows left edge (see Fig. 5). It reduces research to a simple geometrical problem. After calculations we will receive, that at $\arctan \frac{h+\sqrt{r^2-\frac{1}{4}}}{\frac{1}{2}-a} > \varphi > \arctan \frac{h+\sqrt{r^2-\frac{1}{4}}-r}{\frac{1}{2}-a}$:

$$\frac{x}{1-2a} = 1 - \frac{\sin 2(\varphi_1 - \varphi)}{(1-2a)\sin(2\varphi_1 - \varphi)\sin\varphi}\left(h + \sqrt{r^2 - \frac{1}{4}} - r\sin\varphi_1\right) \quad (1)$$

Where φ_1 it is set implicitly by the equation:

$$\left(\frac{1}{2} - a\right)\sin(2\varphi_1 - \varphi) + r\sin(\varphi_1 - \varphi) + \left(h + \sqrt{r^2 - \frac{1}{4}}\right)\cos(2\varphi_1 - \varphi) = 0$$

If an angle of falling bunch is $\varphi > \arctan \frac{h+\sqrt{R^2-\frac{1}{4}}}{\frac{1}{2}-a}$ than right one of beams which are passing through an entrance window does not leave billiard after one collision any more. Taking this into account, for a

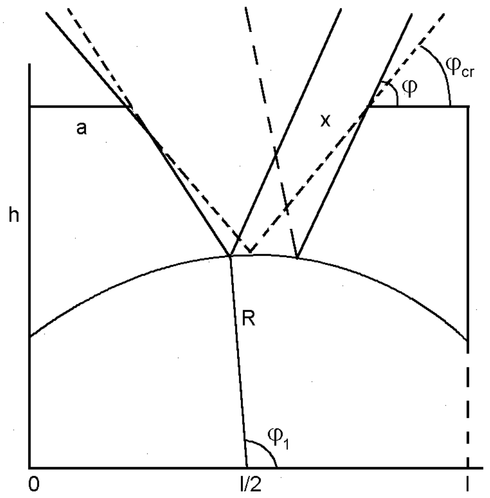

Fig. 5. The bunch of trajectories leaving billiard after one collision with its border. This bunch always contains a direction of the mirror reflection, corresponding to reflection from the central point of a concave segment. By a dotted line the trajectory defining a critical angle is shown.

part of the energy of a bunch leaving after one reflection when $\varphi > \arctan\frac{h+\sqrt{R^2-\frac{1}{4}}}{\frac{1}{2}-a}$ other analytical dependence is received:

$$\frac{x}{1-2a} = 1 - \frac{\sin 2(\varphi_1-\varphi)}{(1-2a)\sin(2\varphi_1-\varphi)\sin\varphi}\left(h+\sqrt{R^2-\frac{1}{4}}-R\sin\varphi_1\right) -$$

$$-\left(\frac{1}{2}-\frac{h+\sqrt{R^2-\frac{1}{4}}}{(1-2a)\tan\varphi_1}\right)\left(1+\frac{\sin(2\varphi_1-\varphi)}{\sin\varphi}\right) \quad (2)$$

The received relationships (1), (2) define a part of energy of beams leaving billiard after one collision in all range of directions of the falling initial bunch. It is possible to prove, that at $\varphi = \arctan\frac{h+\sqrt{r^2-\frac{1}{4}}}{\frac{1}{2}-a}$ the maximum of dependence of energy of the considered reflected bunch is reached. The received analytical dependence (1) is difficult enough so for visual demonstration we will show this dependence at Fig. 6.

The bunch leaving after one reflection is one of the most essential on indicatrix of dispersion and exists for any admitable choice of parameters of billiard. It can be used for establishment of some qualitative criterion, and with its help it is possible to break all of billiards in the considered class into two subclasses: billiards with purely diffusive indicatrix of dispersion and billiards with mixed indicatrix of dispersion, containing more or the less obviously expressed directions of targeted reflection. As billiards with diffusive indicatrix of dispersion we will consider billiards, which energy density of a bunch leaving after one reflection $\frac{p_{max}}{2(\varphi_1-\varphi)}$ at angle of incidence when energy of this bunch has its maximum, much less than the corresponding density of energy in case of purely diffusive reflection type $\frac{1-2a}{\pi}$.

The received relations (1), (2) also define an energy part of beams close to mirror reflection and are in agreement with results of numerical simulation. Similarly, it is possible to estimate analytically an energy part of the bunch of the beams, a leaving billiard for two collisions and so on. These beams also form corresponding splashes in indicatrix of dispersion. The bunches corresponding to high numbers of reflections can be found numerically.

However, let's pass to more important question. We will pay attention, that indicatrix of dispersion has the essential contribution from return dispersion (see Fig. 3). Presence of such type of dispersion is characteristic for superconducting systems with Andreev reflection type [7], but is not observed in usual crystals. Indicatrix of dispersion in scintillation crystals contains only mirror and diffuse parts [6], without

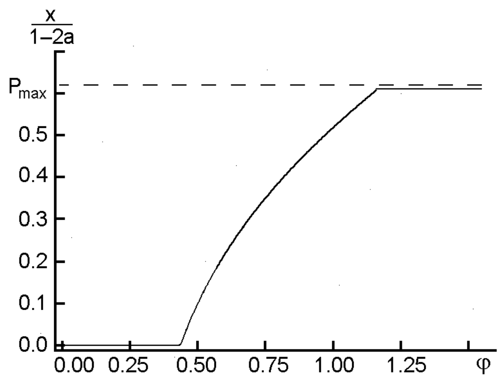

Fig. 6. Dependence of the part of energy of falling beams, corresponding to the bunch leaving after one collision, from angle of incidence $0 \le \varphi \le \pi/2$, for billiard with parameters $h = 0.4$, $r = 0.63$, $a = 0.17$.

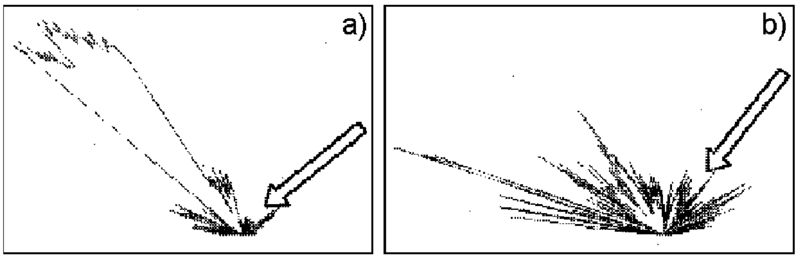

Fig. 7. Indicatrixes of dispersion for microscopic billiards without lateral walls with parameters a) $h = 0.66$, $r = 1$, $a = 0.3$ and b) $h = 0.66$, $r = 1$, $a = 0.46$. By an arrow the direction of falling bunch of beams is shown.

reflection in the opposite direction. For elimination of this effect we will use the possibility of management of the form of microscopic billiard. First of all we will notice, that reflection in the opposite direction is a consequence of reflection from lateral walls of a microscopic billiard. Therefore for reflection modeling in scintillation crystals it is necessary to modify considerably microscopic billiard, having lateral walls removed. From the physical point of view it means change of topology from billiards with transition to billiards on the cylinder. Such billiard can be received sticking together lateral walls with each other. Typical indicatrix of reflections of the modified billiard received numerically is shown in Fig. ??. It is visible, that such indicatrix has corresponding maximum in the direction of mirror reflection. Analytical parities (1), (2) and dependence (Fig. 6) from the angle of incidence for the part of energy of unitary reflection remains the same for a billiard on the cylinder. It is connected with absence of influence of lateral borders on these beams. Thus, the mirror part of reflection appears to have two pieces: reflection from the top flat part of border of billiard and the mirror reflection generated by the interior of microscopic billiard. In this way the part of energy of the falling bunch, reflected in the mirror direction appears to be dependent on an angle of incidence. It is necessary to notice, that presence of flat sites between entrance windows of microscopic billiards is not necessary. Thanks to topology of a microscopic billiards it is possible to make all border of a billiard only from windows. Basically it does such border even more realistic than it is in case of presence of periodic structure of flat sites between entrance windows. In this case mirror component will mainly be defined by beams with unitary reflection. Thus the energy part of mirror component will be defined by the relation $x/(1-2a)$. Thus it is shown, that for modeling of indicatrix of dispersion qualitatively corresponding to observed in scintillation crystals, it is necessary to use microscopic billiards of certain topological type. Selection of quantitative characteristics needs to be carried out after detailed research of influence of change of characteristic parameters of such billiards on indicatrix of dispersion.

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Моделювання дифузійного відбиття за допомогою більярду

Д.М.Наплеков, А.В.Тур, В.В.Яновський

У роботі запропонований метод моделювання дифузно-дзеркального відбиття променів використовуючи мікроскопічні більярди. Показано, що таке моделювання можливо при використанні топологічно незвичайних більярдів на циліндрі які просіюють промені. Виявлено якісні зміни індикатриси розсіювання зі зміною параметрів, що характеризують форму більярду.