

Nonequilibrium thermodynamics of heat radiation conduction in dielectric media

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In framework of representation of fluctuation electromagnetic field, it is built the stochastic model which describes nonequilibrium thermodynamics of heat radiation conduction in paramagnetic dielectric media. For its construction it is used stochastic field of spatially distributed thermodynamic fluctuations of electrical polarization. Its stochastic dynamics is such that the fluctuation-dissipation theorem is fulfilled.

В рамках представления о флуктуационном электромагнитном поле построена стохастическая модель, описывающая неравновесную термодинамику радиационно-кондуктивного теплообмена в парамагнитных диэлектрических средах. В её основе лежит стохастическое поле пространственно распределённых термодинамических флуктуаций спонтанной электрической поляризации, динамика которой подчинена флуктуационно-диссипационной теореме.

1. Introduction

Concrete calculations of heat transfer in semi-transparent solid media when the mechanism of heat radiation conduction is taken into account are based ordinary on the so-called heat transfer theory. This theory is based on representations of geometrical optics at the combination with the Kirchho« law. The last controls heat absorption and its re-emission in media under consideration in red and infra-red regions of spectrum which transfers the heat [1].

At the same time, the successive microscopic theory of heat radiation conduction is absent now. It is connected with the deep reason, i.e. the mechanism of pumping-over of electromagnetic radiation into thermal phonons in solids and the opposite process of electromagnetic waves emission by phonons are manifested e«ectively only due to account of strong bond between medium molecules (ions).

In the meantime, from one side, there are physical situations when deviations from geometrical optics are essential (for example, it is the heat conduction in strong overheated specimens with large linear sizes and large optical transparency), and, from other side, there are media which are constructed very complicated microscopically. In the last case, the study of absorption and irradiation of electromagnetic waves transferring the heat on the basis only the Kirchho« law is inadequate. Due to this reason, it is important such a development of the heat radiation conduction that the electromagnetic »eld satisfying Maxwell's equations in the medium under consideration is introduced explicitly into the theory. In the framework of such a theory, there exists the possibility to solve the theoretical problem connected with two situations above-pointed out.

The base of such a theory has been proposed in Rytov's works [2,3]. The theory was based on the representation of thermal fluctuations of electromagnetic field in the medium. They are generated by thermal oscillations of medium space structure.

However, it has been turned up that it is necessary further adjustment of studied physical situation for explicit calculation of main characteristics of heat radiation conduction. It is the density $\mathbf{S}(\mathbf{r},t)$ of energy flow of thermal electromagnetic field in space point with the radius vector $\mathbf{r}$ which is performed in the form of the functional $\mathbf{S}(\mathbf{r},t) = \mathbf{S}[T(\mathbf{r},t)]$ on the instant temperature distribution $T(\mathbf{r},t)$ in the medium at same time moment t. In other words, it is necessary some additional information about physical nature of the medium. Just on the basis of the functional $\mathbf{S}[T(\mathbf{r},t)]$, the following self-consistent evolution equation describing the heat transfer in the medium is built

$$C(T)\dot{T} = (\nabla, \varkappa(T)\nabla T) - (\nabla, \mathbf{S}), \tag{1}$$

where $C(T)$ is the medium specific heat and $\varkappa(T)$ is its the thermal conduction coefficient. In particular, such calculations have been done by authors [4, 5] in the case when the medium is dielectric or it is high-resistive semiconductor with covalent chemical bond. In that case, there are no (they are strongly depressed) fluctuations of electrical charge from the microscopic viewpoint and, besides, there are no some fluctuations of electrical current. The Rytov theory operates namely these values [2, 3]. Since the spin of medium molecules is equal zero, it permits to suppose also the strong depression of microfluctuations of magnetic moment density in the medium. In works [4, 5], it is supposed also that the medium is isotropic from the electrodynamic viewpoint and the influence of the spatial dispersion of electromagnetic waves spreading is negligibly small. The theory having developed in cited works is turned up in complete correspondence with the heat transfer theory when the geometrical optics approximation is applicable from the physical viewpoint. But all calculations in works pointed out are oriented by most degree way in order to find the parameter region where the classic theory of heat transfer is applicable. General formulation of theoretical approach to description of heat radiation conduction on the basis of main concepts of fluctuation theory does not considered.

2. Gaussian field of electrical polarization

We consider the fluctuation electromagnetic field $\mathbf{E}(\mathbf{r},t)$, $\mathbf{H}(\mathbf{r},t)$ in the medium of such type that is studied in works [4, 5]. Fields $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ satisfy the system of Maxwell equations

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -[\nabla, \mathbf{E}]\,, \quad \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = [\nabla, \mathbf{H}]\,, \tag{2}$$

$$(\nabla, \mathbf{B}) = 0\,, \quad (\nabla, \mathbf{D}) = 0\,, \tag{3}$$

where we put $\mathbf{B}(\mathbf{r},t) = \mu\mathbf{H}(\mathbf{r},t)$ with constant magnetic permeability μ according to above-mentioned medium nature. and $\mathbf{D}(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t) + 4\pi\mathbf{P}(\mathbf{r},t)$, $\mathbf{P}(\mathbf{r},t)$ is the electrical polarization vector in the space point with the radius vector $\mathbf{r}$. We put the material equation for $\mathbf{P}(\mathbf{r},t)$, using the linear form of electrodynamics, in the form

$$\mathbf{P}(\mathbf{r},t) = \int_{-\infty}^{\infty} \hat{\chi}(t-t')\mathbf{E}(\mathbf{r},t')dt' + \tilde{\mathbf{P}}(\mathbf{r},t)\,, \tag{4}$$

where $\hat{\chi}(t)$ is instant dynamical electrical medium susceptibility. It satisfy $\hat{\chi}(t) = 0$ at $t < 0$. Eq.(4) describes the linear medium reaction on the perturbation by electrical field where we take into account the negligible smallness of spatial dispersion, the medium isotropy and the absence of magnetoelectrical effect in it. The function $\tilde{\mathbf{P}}(\mathbf{r},t)$ describes inherent electrical medium polarization. The fact that we restrict ourselves only linear part of the medium reaction is connected with that electrical field $\mathbf{E}(\mathbf{r},t)$ has the thermal origination and, therefore, it is small. Inherent electrical polarization, in our case, describes thermal random fluctuations of this physical characteristics which occur at the absence of external perturbation. Just these fluctuations are the source of thermal electromagnetic radiation in Eqs.(1),(2).

Stochastic character of the function $\tilde{\mathbf{P}}(\mathbf{r},t)$ is connected not only with the fact that it is the characteristic value of system with large number of particles, i.e. medium molecules located near the point with the radius vector $\mathbf{r}$, but it is connected with the quantum nature of irradiation and absorption of thermal photons.

Introducing the dynamical medium electrical permeability

$$\hat{\varepsilon}(t) = \delta(t) + 4\pi\hat{\chi}(t)$$

such that $\hat{\varepsilon}(t) = 0$ at $t < 0$, we write down, on the basis of (2), (3), the complete equation system describing Áuctuation electromagnetic »eld

$$\frac{\mu}{c}\frac{\partial \mathbf{H}}{\partial t} = -[\nabla, \mathbf{E}]\,, \quad \frac{1}{c}\frac{\partial}{\partial t}\int\limits_{-\infty}^{\infty} \hat{\varepsilon}(t-t')\mathbf{E}(\mathbf{r},t')dt' + \frac{4\pi}{c}\tilde{\mathbf{j}} = [\nabla, \mathbf{H}]\,, \tag{5}$$

$$(\nabla, \mathbf{H}) = 0\,, \quad \int\limits_{-\infty}^{\infty} \hat{\varepsilon}(t-t')(\nabla, \mathbf{E})(\mathbf{r},t')dt' = 4\pi\tilde{\rho}\,, \tag{6}$$

where the e«ective electrical current density $\tilde{\mathbf{j}}(\mathbf{r},t) = \dot{\tilde{\mathbf{P}}}(\mathbf{r},t)$ and the e«ective electrical charge density $\tilde{\rho}(\mathbf{r},t) = -(\nabla, \tilde{\mathbf{P}})(\mathbf{r},t)$ are introduced which satisfy the continuity equation

$$\frac{1}{c}\frac{\partial\tilde{\rho}}{\partial t} + (\nabla, \tilde{\mathbf{j}}) = 0$$

according to their de»nition.

Here, it shows up the di«erence of developed approach from that has been performed in [3] where relatedness of Áuctuation currents and charges is not claimed.

Our next problem consists of the building of random function $\tilde{\mathbf{P}}(\mathbf{r},t)$ such that its probability distribution functionally depends on instant temperature distribution in the medium. It is necessary do this on the basis of reason suppositions. Firstly, it is necessary to put that the average value $\langle\tilde{\mathbf{P}}(\mathbf{r},t)\rangle$ of electrical polarization Áuctuations is equal to zero. Secondly, due to Áuctuation smallness, one may be restricted the gaussian model of random »eld $\tilde{P}_k(\mathbf{r},t)$, $k = 1,2,3$. Therefore, for complete characterization of random »eld $\tilde{P}_k(\mathbf{r},t)$, it is suБcient to point out its correlation function. Though, this correlation function must carry the information about the instant temperature distribution $T(\mathbf{r},t)$. Therefore, the »eld $\tilde{P}_k(\mathbf{r},t)$ cannot be uniform. With the aim of construction of this spatially nonuniform and not temporal stationary gaussian »eld, we consider its physical reason.

Since the correlation radius of Áuctuations $\tilde{\mathbf{P}}(\mathbf{r},t)$ has the scale of average distance between molecules of the order value and the spreading of thermal photons occurs on the distance which is many more of this value (the medium is semitransparent and it has no very large absorption of electromagnetic radiation in the spectrum region being important for heat radiation conduction), then one may be neglect space correlations of Áuctuations and be consider that $\langle\tilde{P}_k(\mathbf{r},t)\tilde{P}_{k'}(\mathbf{r}',t')\rangle \sim \delta_{kk'}\delta(\mathbf{r}-\mathbf{r}')$. Here, we have taken into account also the stochastic isotropy of correlation function by the Kronecker symbol $\delta_{kk'}$.

If the random swinging of electrical polarization should not be connected with irradiation and absorption of photons, one may be consider also that its temporal correlation function has the form of δ-function. In order to show real character of temporal correlations appearing due to transit of photons and show the character of spatial and temporal nonuniformity of the »eld $\tilde{P}_k(\mathbf{r},t)$, we take into account the Áuctuations $\tilde{\mathbf{P}}(\mathbf{r},t)$ connected with molecules being near the point with radius vector $\mathbf{r}$ which irradiate and absorb photons. Then this random function naturally to connect with photon distribution irradiating this molecule system.

Photons with di«erent frequencies represent themselves ideal gas and therefore they do not correlate with each other. Correspondingly, the frequency correlation function of spectral amplitudes

$$\bar{P}_k(\mathbf{r},\omega) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \tilde{P}_k(\mathbf{r},t)e^{-i\omega t}dt\,, \tag{7}$$

should be proportional to $(2\pi)^{-1}\delta(\omega-\omega')\delta(\mathbf{r}-\mathbf{r}')$ in this case. Let us »nd the coeБcient of the proportionality for this δ-function, i.e. the spectral density.

Since the physical dimensionality of the function $\tilde{P}_k$ is equal to $\mathrm{T}^{-1}(\mathrm{M/L})^{1/2}$ and, consequently, the dimensionality of $\bar{P}_k$ is equal $(\mathrm{M/L})^{1/2}$, then the dimensionality of spectral density is equal T·[energy]/[volume]. Then it is natural to connect the desired spectral density with the spectral density of photons with the frequency ω which are in the volume unit.

Introducing the frequency photon density $\tau W(\hbar\omega/\kappa T(\mathbf{r},t))$ normalized on unit. It is the density at the irradiator temperature $T(\mathbf{r},t)$ where τ is temporal multiplier equal numerically to average time of energy transitions in molecules, we represent the spectral density by the expression $\tau\hbar\omega W(\hbar\omega/\kappa T(\mathbf{r},t))$. Thus, one may write down

$$\langle \tilde{P}_k(\mathbf{r},\omega)\tilde{P}^*_{k'}(\mathbf{r}',\omega')\rangle = \frac{\tau}{2\pi}\hbar\omega W\left(\frac{\hbar\omega}{\kappa T(\mathbf{r},t)}\right)\delta_{kk'}\delta(\omega-\omega')\delta(\mathbf{r}-\mathbf{r}')\,. \tag{8}$$

Then one may put formally

$$\tilde{P}_k(\mathbf{r},\omega) = U(T(r,t),\omega)\varphi_k(\mathbf{r},\omega)\,, \tag{9}$$

where $U(T,\omega) = [\tau\hbar\omega W(\hbar\omega/\kappa T)]^{1/2}$ and $\varphi_k(\mathbf{r},\omega)$ is a "standard" generalized gaussian random function with zero average value and the correlation function

$$\langle \varphi_k(\mathbf{r},\omega)\varphi^*_{k'}(\mathbf{r}',\omega')\rangle = \frac{1}{2\pi}\,\delta_{kk'}\delta(\omega-\omega')\delta(\mathbf{r}-\mathbf{r}')\,. \tag{10}$$

In terms of this random function, the current and the charge of Auctuations are expressed according to (7) by formulas

$$\tilde{j}_k(\mathbf{r},t) = \int\limits_{-\infty}^{\infty}\left(i\omega U(T(\mathbf{r},t),\omega) + \frac{\partial}{\partial t}U(T(\mathbf{r},t),\omega)\right)\varphi_k(\mathbf{r},\omega)e^{i\omega t}d\omega\,, \tag{11}$$

$$\tilde{\rho}(\mathbf{r},t) = -\int\limits_{-\infty}^{\infty}\left(U(T(\mathbf{r},t),\omega)\nabla_j\varphi_j(\mathbf{r},\omega) + \varphi_k(\mathbf{r},\omega)\nabla_k U(T(\mathbf{r},t),\omega)\right)e^{i\omega t}d\omega\,. \tag{12}$$

3. Quasistationary thermal electromagnetic »eld

Now, we show how the main problem of heat radiation conduction should be solved in framework of proposed stochastic model. It consists of the calculation of the average $\langle(\nabla,\mathbf{S}(\mathbf{r},t))\rangle$ at arbitrary temperature distribution where the energy Аow density $\mathbf{S}$ is de»ned by the formula

$$\mathbf{S}(\mathbf{r},t) = \frac{c}{4\pi}[\mathbf{E},\mathbf{H}](\mathbf{r},t)\,. \tag{13}$$

Thus, at the value $\langle(\nabla,\mathbf{S}(\mathbf{r},t))\rangle$ calculation it should be produce the averaging of expression quadratic on random gaussian »elds in righthand side of this formula. Fields $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ are gaussian due to linearity of equation system (5),(6) and also the linearity of expressions (11), (12). This average is calculated on the basis of explicit form of linear transformations of the random »eld $\varphi_k(\mathbf{r},\omega)$. Application these transformations to the »eld $\varphi_k(\mathbf{r},\omega)$ gives expressions of »elds $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$. The form of transformations is de»ned by the Green function of the initial boundary problem with conditions on boundary of medium region. These conditions reАect the continuity of »eld $\mathbf{E}(\mathbf{r},t)$, $\mathbf{H}(\mathbf{r},t)$ passage with their tangential derivatives into the region out of medium. In the external region, these »elds are solutions of vacuum Maxwell's equations. They should be have the asymptotical form of diverging spherical waves at unbounded removing from the medium region. Initial conditions of desired solutions are put equal to zero that reАect the absence of deterministic components. In a result, the average $\langle(\nabla,\mathbf{S}(\mathbf{r},t))\rangle$ takes the form of integral transformation to the average $\langle\tilde{P}_k(\mathbf{r},\omega)P^*_{k'}(\mathbf{r}',\omega')\rangle$. It depends by nonlinear way on the temperature distribution $T(\mathbf{r},t)$. The nonlinearity form is de»ned by the form of distribution $W(\cdot)$. Now, we realize this averaging procedure explicitly when the corresponding Green function is known.

We introduce special vector-functions $E_j(\mathbf{r},t,\omega)$, $H_j(\mathbf{r},t,\omega)$, $j=1,2,3$ depending on the frequency ω which satisfy the following equation system

$$\nabla_k H_k(\mathbf{r},t,\omega)=0\,,\qquad \varepsilon(\omega)\nabla_k E_k(\mathbf{r},t,\omega)=4\pi\rho(\mathbf{r},t,\omega)\,, \tag{13}$$

$$\rho(\mathbf{r},t,\omega)=-U(T(\mathbf{r},t),\omega)\nabla_j\varphi_j(\mathbf{r},\omega)-\varphi_j(\mathbf{r},\omega)\nabla_j U(T(\mathbf{r},t),\omega)\,, \tag{14}$$

$$\frac{\mu}{c}\frac{\partial}{\partial t}H_k(\mathbf{r},t,\omega)+i\frac{\omega\mu}{c}H_k(\mathbf{r},t,\omega)=-\epsilon_{klm}\nabla_l E_m(\mathbf{r},t,\omega)\,,\quad k=1,2,3\,, \tag{15}$$

$$\frac{\varepsilon(\omega)}{c}\frac{\partial}{\partial t}E_k(\mathbf{r},t,\omega)+i\frac{\omega}{c}\varepsilon(\omega)E_k(\mathbf{r},t,\omega)+\frac{4\pi}{c}j_k(\mathbf{r},t,\omega)=\epsilon_{klm}\nabla_l H_m(\mathbf{r},t,\omega)\,,\quad k=1,2,3\,, \tag{16}$$

$$j_k(\mathbf{r},t,\omega)=\varphi_k(\mathbf{r},\omega)\left(i\omega U(T(\mathbf{r},t),\omega)+\frac{\partial}{\partial t}U(T(\mathbf{r},t),\omega)\right)\,,\quad k=1,2,3 \tag{17}$$

where the symbol ϵ_{klm}, $k,l,m=1,2,3$ represents the Levi-Chivita pseudotensor. On its physical sense, introducing functions describe »elds being quasistationary relative to fast process of electromagnetic radiation spreading in specimen.

Solutions of original problem are built on the basis of solutions of the system (13)-(17) with zero initial conditions at $t=0$ for »elds $E_j(\mathbf{r},t,\omega)$, $H_j(\mathbf{r},t,\omega)$, $j=1,2,3$ and, besides, satisfying boundary conditions in the form of »eld continuous transition and also continuous transition of their tangential derivatives on region boundary into corresponding »elds out of the region. Last »elds satisfy Maxwell's equations in vacuum and they have the asymptotical form of spherical waves diverging at in»nity. Namely, one may put

$$\mathbf{E}(\mathbf{r},t)=\int_{-\infty}^{\infty}e^{i\omega t}\mathbf{E}(\mathbf{r},t,\omega)d\omega\,,\qquad \mathbf{H}(\mathbf{r},t)=\int_{-\infty}^{\infty}e^{i\omega t}\mathbf{H}(\mathbf{r},t,\omega)d\omega \tag{18}$$

according to Eqs. (5), (6), (11), (12).

In terms of introducing functions, the energy Aow density is expressed by the formula

$$S_k(\mathbf{r},t)=\frac{c}{4\pi}\int_{-\infty}^{\infty}e^{i\omega t}\left(\int_{-\infty}^{\infty}\epsilon_{klm}E_l(\mathbf{r},t,\omega')H^*_m(\mathbf{r},t,\omega'-\omega)d\omega'\right)d\omega\,. \tag{19}$$

From equations (13)-(17), some separate equations for »elds $\mathbf{E}(\mathbf{r},t,\omega)$ and $\mathbf{H}(\mathbf{r},t,\omega)$ are obtained,

$$\ddot{H}_k+2i\omega\dot{H}_k-\omega^2H_k+\frac{c^2}{n^2(\omega)}\Delta H_k=\frac{4\pi c}{n^2(\omega)}\epsilon_{klm}\nabla_l j_m\,,$$

$$\ddot{E}_k+2i\omega\dot{E}_k-\omega^2E_k+\frac{c^2}{n^2(\omega)}\Delta E_k=-\frac{4\pi}{\varepsilon(\omega)}\left(i\omega j_k+\frac{\partial j_k}{\partial t}+\frac{c^2}{n^2(\omega)}\nabla_k\rho\right)\,,$$

$n^2(\omega)=\mu\varepsilon(\omega)$. They di«er from each to other only by sources.

Let $G(\mathbf{r}-\mathbf{r}',t-t';\omega)$ be the Green function of initial boundary problem with desired conditions on the region boundary, i.e.

$$\frac{\partial^2 G}{\partial t^2}+2i\omega\frac{\partial G}{\partial t}-\omega^2 G+\frac{c^2}{n^2(\omega)}\Delta G=\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\,. \tag{20}$$

Then

$$H_k(\mathbf{r},t,\omega'-\omega)=\frac{4\pi c}{n^2(\omega'-\omega)}\epsilon_{klm}\int_{\Omega}\left(\int_0^t G(\mathbf{r}-\mathbf{r}',t-t';\omega'-\omega)\nabla'_l j_m(\mathbf{r}',t',\omega'-\omega)dt'\right)d\mathbf{r}'\,,$$

$$E_k(\mathbf{r},t,\omega')=$$

$$= -\frac{4\pi}{\varepsilon(\omega')}\int\limits_{\Omega}\left(\int\limits_{0}^{t} G(\mathbf{r}-\mathbf{r}'',t-t'';\omega')\left(\frac{\partial j_k}{\partial t''}+i\omega' j_k+\frac{c^2}{n^2(\omega')}\nabla''_k\rho\right)(\mathbf{r}'',t'',\omega')dt''\right)d\mathbf{r}''$$

where Ω is the internal region occupied by medium.

Averaging on Auctuations gives

$$\langle S_j(\mathbf{r},t,\omega)\rangle \equiv \frac{c}{4\pi}\int\limits_{-\infty}^{\infty}\epsilon_{klm}\langle E_l(\mathbf{r},t,\omega')H^*_m(\mathbf{r},t,\omega'-\omega)\rangle d\omega' =$$

$$= -\int\limits_{-\infty}^{\infty}\frac{4\pi c^2\epsilon_{jkl}\epsilon_{lmn}}{n^2(\omega')\varepsilon(\omega')}d\omega'\int\limits_{\Omega}\left(\int\limits_{0}^{t} G(\mathbf{r}-\mathbf{r}'',t-t'';\omega')\times\right.$$

$$\left.\times\nabla'_m\left[\int\limits_{\mathbb{R}^3}\left[\int\limits_{0}^{t} G(\mathbf{r}-\mathbf{r}',t-t';\omega')\langle u_k(\mathbf{r}'',t'',\omega')j^*_n(\mathbf{r}',t',\omega'-\omega)\rangle\,dt''\right]d\mathbf{r}''\right]dt'\right)d\mathbf{r}' \qquad (21)$$

where

$$u_k(\mathbf{r}'',t'',\omega') = \frac{\partial j_k}{\partial t''}+i\omega' j_k+\frac{c^2}{n^2(\omega')}\nabla''_k\rho \qquad (22)$$

and it has taken into account that the averaging expression $\langle u_k\, j^*_n\rangle$ under the integral sign is proportional $\delta(\omega)$. Generally, it represents bulky expression and it contains terms of di«erent value order. Its simpli»cation is connected with extraction only those terms which give main contribution into averaging divergence of energy Aow density when the self-consistent equation for temperature distribution is built.

First simpli»cation is connected with the notice that it may be neglect the term with the derivative $\partial U/\partial t$ in the expression in the formula (17) of current j_k. It may do on comparison with »rst term, since temperature distribution is changed temporally by very slow way, and the multiplier ω in »rst term is very large. Therefore, we put

$$j_k(\mathbf{r},t,\omega) = i\omega U(T(\mathbf{r},t),\omega)\varphi_k(\mathbf{r},\omega)\,,\quad k=1,2,3\,. \qquad (23)$$

Due to analogous reason the term $\partial j_k/\partial t''$ in Eq.(22) is droped. The average $\langle u_k\, j^*_n\rangle$ is concentrated in hyperplane where $\mathbf{r}'=\mathbf{r}''$. It means that each its term is proportional δ-function $\delta(\mathbf{r}'-\mathbf{r}'')$ or its derivatives. Indeed, on the basis of the above, taking into account Eq.(23), we have

$$\langle u_k j^*_n\rangle \approx i\omega'\langle j_k(\mathbf{r}'',t'',\omega')j^*_n(\mathbf{r}',t',\omega'-\omega)\rangle + \frac{c^2}{n^2(\omega')}\nabla''_k\langle\rho(\mathbf{r}'',t'',\omega')j^*_n(\mathbf{r}',t',\omega'-\omega)\rangle =$$

$$= \delta(\omega)\frac{i\omega'}{2\pi}\left(\omega'^2U'U''\delta_{kn}\delta(\mathbf{r}''-\mathbf{r}')+\frac{c^2}{n^2(\omega')}\nabla''_kU'\left[U''\nabla_n\delta(\mathbf{r}''-\mathbf{r}')+\delta(\mathbf{r}''-\mathbf{r}')\nabla''_nU''\right]\right) \qquad (24)$$

where $U''=U(T(\mathbf{r}'',t''),\omega')$, $U'=U(T(\mathbf{r}',t'),\omega')$.

Since the Green function is spatially changed by fast way in comparison with the temperature distribution $T(\mathbf{r},t)$ due to the large multiplier before the laplacian in Eq.(20) then main terms in the integral on $\mathbf{r}'$ (after calculation of the integral on $\mathbf{r}''$ with the help of δ-function) are those where spatial derivatives of the Green function have most large order. Terms having most large order of derivative from the function $\delta(\mathbf{r}''-\mathbf{r}')$ in Eq.(24) correspond to them. Therefore, at accepted approximation, »rst term in Eq.(24) should be threw out. Further, having acted by the operator ∇'_m on the expression in Eq.(24) which is under the action of the operator ∇''_k, and, after that, having reduced the obtained result with the Levi-Chivita symbol ϵ_{lmn}, we obtain

$$\epsilon_{lmn}\left(U''(\nabla'_mU')\nabla''_n\delta(\mathbf{r}''-\mathbf{r}')+U'(\nabla''_nU'')\nabla'_m\delta(\mathbf{r}''-\mathbf{r}')\right)\,.$$

Consequently, using the identity $\epsilon_{jkl}\epsilon_{lmn}=\delta_{jm}\delta_{kn}-\delta_{jn}\delta_{km}$, we have

$$\epsilon_{jkl}\epsilon_{lmn}\nabla'_m\langle u_kj^*_n\rangle \approx \delta(\omega)\frac{i\omega' c^2}{2\pi n^2(\omega')}\,\times$$

$$\times\ \nabla''_k \left(\left[U'\nabla''_k U'' + U''\nabla'_k U' \right] \nabla'_j \delta(\mathbf{r}'' - \mathbf{r}') - \left[U'\nabla''_j U'' + U''\nabla_j U' \right] \nabla'_k \delta(\mathbf{r}'' - \mathbf{r}') \right) .$$

Substitution of obtained expression into the formula (19) with account of Eq.(21) gives

$$\langle S_j(\mathbf{r},t)\rangle = 2\tau\hbar c^4 \cdot \mathrm{Im} \int\limits_{-\infty}^{\infty} \frac{\omega d\omega}{\varepsilon(\omega) n^4(\omega)} \times$$

$$\times \int\limits_{\Omega} \left(\int\limits_0^t G(\mathbf{r}-\mathbf{r}', t-t';\omega) \nabla_k S_{jk}(\mathbf{r},t;\mathbf{r}',t';\omega) dt' \right) d\mathbf{r}' \qquad (25)$$

where

$$S_{jk}(\mathbf{r},t;\mathbf{r}',t';\omega) =$$

$$= \nabla_k \int\limits_0^t G(\mathbf{r}-\mathbf{r}', t-t'';\omega) V_j(\mathbf{r}',t',t'';\omega) dt'' - \nabla_j \int\limits_0^t G(\mathbf{r}-\mathbf{r}', t-t'';\omega) V_k(\mathbf{r}',t',t'';\omega) dt'' , \qquad (26)$$

$$V_j(\mathbf{r}',t',t'';\omega) =$$

$$= W^{1/2}\left(\frac{\hbar\omega}{\kappa T(\mathbf{r}',t')} \right) \nabla'_j W^{1/2}\left(\frac{\hbar\omega}{\kappa T(\mathbf{r}',t'')} \right) + W^{1/2}\left(\frac{\hbar\omega}{\kappa T(\mathbf{r}',t'')} \right) \nabla'_j W^{1/2}\left(\frac{\hbar\omega}{\kappa T(\mathbf{r}',t')} \right) . \qquad (27)$$

Formulas (25)-(27) solve the problem of general expression of energy flow density of thermal electromagnetic field which has been averaged on fluctuations. They express this value in terms of Green function G of initial boundary problem of differential equation (20) and also in terms of temperature distribution depended on photon distribution density $W\left(\frac{\hbar\omega}{\kappa T}\right)$. The initial boundary problem is connected with the description of radiating point source.

4. Conclusion.

In the work the theoretical model of heat radiation conduction in semitransparent dielectrics is investigated. This model has been proposed on the basis of the concept of electromagnetic radiation generated by thermal fluctuations of electric polarization. Though the obtained expression is rather complicated, however, it reduces the problem of calculation of averaged energy flow density corresponding to thermal electromagnetic field to the calculation of the Green function of standard initial boundary problem for hyperbolic equation with constant coefficients. We notice that representations of geometrical optics (the eikonal approximation in electrodynamics) are not used at the obtaining of formulas (25)-(27). Then the expression obtained is valuable in very wide diapason of physical parameters varying.

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Нерівноважна термодінаміка радіаційно-кондуктивного теплообміну у діелектричних середовищах

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У рамках уявлення про флуктуаційне електромагнітне поле побудовано стохастична модель, що описує нерівноважну термодінаміку радіаційно-кондуктивного теплообміну у парамагнітних діелектричних середовищах. В її основі лежить стохастичне поле просторово розподілених термодінамічних флуктуацій спонтанної електричної поляризації, дінаміка якої підлягає флуктуаційно-дісипативній теоремі.