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# Transverse magnetic mode nonlinear waveguide slab optical sensor utilizing left-handed materials

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In the study it was considered that slab waveguide structure has a left-handed material (LHM) as a guiding layer. Propagation of transverse magnetic modes in the structure for homogeneous optical sensing applications was investigated theoretically. The LHM is embedded between a semi-infinite linear substrate and a semi-infinite nonlinear cladding with an intensity-dependent refractive index of Kerr type. The investigation was focused on the sensitivity of the proposed optical sensor to provide the designer with optimum structure that corresponds to the highest sensitivity.

В работе рассматривается пластинчатая волноводная структура, которая содержит левовращающий материал (ЛВМ) в качестве направляющего слоя. Теоретически исследовано распространение поперечно-магнитных мод в структуре, применяемой для оптического зондирования однородных сред. ЛВМ помещен между полубесконечной оптически линейной подложкой и полубесконечным нелинейным покрытием с зависящим от интенсивности по закону Керра показателем преломления. Исследование в основном направлено на определение чувствительности предлагаемого оптического сенсора для предоставления разработчикам данных об оптимальной структуре сенсора, соответствующей максимальной чувствительности.

## *1. Introduction*

The concept of negative refractive index (RI) materials proposed by Veselago [1] and verified experimentally by Smith [2] has attracted remarkable interest due to demonstration of unusual electromagnetic properties. These materials are known as Metamaterials (MTMs), double negativity (negative $\varepsilon$ and $\mu$) materials or Left-Handed Materials (LHM). The phenomena of negative RI are observed in microwave, millimeter-wave, and optical frequency bands. The LHM is a hand-made structure that can be designed to exhibit specific phenomena not commonly found in nature.

A homogeneous flat slab of a material with negative RI can behave as a perfect lens [3], for which numerous applications in science can be expected. It is shown that left-handed materials can enhance the evanescent field in slab waveguides [4]. In 2006, Sabah et al. [5] studied the effects of structure parameters, incidence angle, and frequency on the reflected and transmitted powers for the lossless LHM structure. Electromagnetic wave propagation through frequency-dispersive and lossy double-negative slab embedded between two different semi-infinite media was presented in 2007 [6]. El-Khozondar et al. [7] used LHM to enhance the performance of Optical Waveguide Isolator.

Optical sensing is a rapidly growing area of research. Some of the main applications are related to evaluation of protein adsorption, affinity-based recognition and attaching bacteria or living cells. The optical waveguide sensors make use of guided modes in slab waveguides for chemical and biological applications [8–10]. In particular, the fundamental modes $TE_0$ and $TM_0$ in very thin planar waveguides of high RI are used in this attractive field. The principle of optical waveguide sensors is as follows. The evanescent field of the guided mode interacts with the sample to be detected (analyte) and it senses changes in the RI of the superstrate (cladding). Thus changes in the effective RI ($N$) of the guided mode are induced. The sensitivity $S$ of the optical waveguide sensor is defined as a ratio of change of the effective RI ($N$) and change of the cladding index $n_c$, i.e., $S = \partial N/\partial n_c$. Taya et al. [11] has showed analytically that the sensitivity of an optical waveguide sensor can be dramatically enhanced by using LHM. In another study, El-Khozondar et al. [12] showed that the behavior of LHM optical waveguide subjected to stress can be controlled by adjusting the double-negative materials parameters.

Authors take a further step to study a new sensor in which their proposed sensor consists of both nonlinear materials and LHM. Using this combination a sensor with enhanced characteristics can be created. The proposed structure consists of thin film composed of LHM with thickness $t$. The thin film is sandwiched between a linear substrate of permittivity $\varepsilon_s$ and a Kerr-type nonlinear cladding of nonlinear permittivity $\varepsilon_{nl}$.

## 2. Theoretical background

In current study $p$-polarized waves that propagate in the $z$-direction (TM waves) were considered. The fields components are $H_y$, $E_x$, and $E_z$. To solve the nonlinear wave equation in the cladding medium, we define $\varepsilon_{nl} = \varepsilon_c + \alpha'|H_y|^2$, where $\alpha' = \alpha/\varepsilon_c c^2 \varepsilon_0^2$, $\alpha$ is the nonlinear coefficient, $\varepsilon_c$ is the linear part of the permittivity, $\varepsilon_0$ is free space permittivity and $c$ is speed of light in vacuum [13, 14]. The LHM in the film is assumed to have negative $\varepsilon_f$ and negative $\mu_f$ and the substrate is linear dielectric media with permittivity $\varepsilon_s$.

The solution of the wave equation in each layer is found to be

$$H_{y1} = \sqrt{\frac{2}{\alpha'}}\,\frac{q_c}{\cosh[k_0 q_c (x + x_0)]}, \quad x > 0, \tag{1}$$

$$H_{y2} = A\cos(k_0 q_f x + B\sin(k_0 q_f x), \quad -t < x < 0, \tag{2}$$

$$H_{y3} = C e^{k_0 q_s (x+t)}, \quad x < -t, \tag{3}$$

where $N$ is the effective refractive index, $k_0$ is the free space wavenumber, $q_f = \sqrt{\frac{\mu_f}{\mu_0}\varepsilon_f - N^2}$, $q_s = \sqrt{N^2 - \varepsilon_s}$, $q_c = \sqrt{N^2 - \varepsilon_c}$, $A$, $B$ and $C$ are constants represent the amplitude of the wave, and $x_0$ is a constant related to the power propagating in the waveguide. The field peaks in the cladding layer at $x = x_0$ [15].

The magnetic field $H_0$ at the clad-film interface is obtained by substituting $x = 0$ in $H_{y1}$-relation in Eq.(1). This gives

$$\frac{\alpha' H_0^2}{2} = q_c^2 (1 - \tanh^2(k_0 q_c x_0)), \tag{4}$$

where $\alpha' H_0^2/2$ is called clad-film interface nonlinearity [16]. The tangential component of the electric field $E_z$ is calculated using equation $E_z = \frac{-j}{\omega\varepsilon}\frac{\partial H_y}{\partial x}$. The electric field $E_z$ in each layer is

$$E_{z1} = \frac{j k_0 q_c}{\varepsilon_0 \varepsilon_c \omega}\sqrt{\frac{2}{\alpha'}}\operatorname{sech}[k_0 q_c (x + x_0)]\times \times\tanh[k_0 q_c (x + x_0)], \quad x > 0, \tag{5}$$

$$E_{z2} = \frac{-j k_0 q_f}{\varepsilon_0 \varepsilon_f \omega}[-A\sin(k_0 q_f x) + B\cos(k_0 q_f x)], \quad -t < x < 0, \tag{6}$$

$$E_{z3} = \frac{-j k_0 q_c}{\varepsilon_0 \varepsilon_c \omega} C e^{k_0 q_s (x+t)}, \ x < -t\ . \tag{7}$$

The continuity requirements of $H_y$ and $E_z$ results in the following dispersion equation

$$k_0 q_f t = \arctan\left(\frac{q_s \varepsilon_f}{q_f \varepsilon_s}\right) + + \arctan\left(\frac{q_c \varepsilon_f}{q_f \varepsilon_c}\tanh(k_0 q_c x_0)\right) + m\pi, \tag{8}$$

where $m = 0, 1, 2, \ldots$ is the mode order.

Sensitivity is defined as the variation of the effective refractive index ($N$) with respect to the changes of the cladding refractive index ($n_c$). Differentiating the dispersion relation given by Eq. (8) with respect to $N$, the sensitivity $S$ of the proposed waveguide sensor to changes in the RI of the cladding is

$$S = \frac{\sqrt{a_c(1+r_c^2)}\left[a_c\kappa + a_c\tanh(k_0q_cx_0) + \frac{2r_c^2\tanh(k_0q_cx_0)}{1+r_c^2}\left(\frac{\mu_f}{\mu_0} - a_c\right)\right]}{r_c\zeta\sqrt{a_c + \frac{\mu_f}{\mu_0}r_c^2}(k_0q_ft + T_s + T_c)}, \tag{9}$$

where $\kappa = k_0q_cx_0\left(1 - \tanh^2(k_0q_cx_0)\right)$,

$$\xi = a_c^2 + r_c^2\tanh^2(k_0q_cx_0),\quad r_c = \frac{q_c}{q_f},\quad r_s = \frac{q_s}{q_f},$$

$$T_c = \frac{a_c\kappa + a_c(1 + r_c^2)\tanh(k_0q_cx_0)}{r_c\xi},$$

$$T_s = \frac{a_s(1 + r_s^2}{r_s(a_s^2 + r_s^2)},\quad a_c = \frac{\varepsilon_c}{\varepsilon_f} \text{ and } a_s = \frac{\varepsilon_s}{\varepsilon_f}.$$

The power flow through the layers of the structure is a critical parameter for the sensitivity of the optical waveguide sensor. For TM-modes the total energy flux per length unit is given by

$$P_{total} = \frac{Nk_0}{2\omega\varepsilon_0\varepsilon_r}\int_{-\infty}^{\infty} H_y^2dx = P_s + P_f + P_c, \tag{10}$$

where $P_s$, $P_f$ and $P_c$ are the energy flux per length unit as calculated for the substrate, film and cladding media, respectively. Then we replace $H_y$ in Eq.(10) by their values as defined by Eq.(1), Eq.(2) and Eq.(3). The fraction of the total power that is flowing into the cladding is given by

$$\frac{P_c}{P_{total}} = \frac{\frac{r_cq_f}{\alpha'\varepsilon_c}y}{\frac{r_cq_f}{\alpha'\varepsilon_c}y + \frac{k_0}{2\varepsilon_f}\left[\frac{A^2}{2}x_- + \frac{B^2}{2}x_+ + \frac{AB}{k_0q_f}\sin^2(k_0q_ft)\right] + \frac{C^2}{4\varepsilon_sr_sq_f}}, \tag{11}$$

where $y = 1 - \tanh(k_0q_cx_0)$,

$$x_- = t - \frac{\sin(2k_0q_ft)}{2k_0q_f} \text{ and } x_+ = t + \frac{\sin(2k_0q_ft)}{2k_0q_f}.$$

The sensitivity of the proposed sensor strongly depends on the parameters of the structure. For a given configuration with constant $\varepsilon_c$, $\varepsilon_f$, and $\varepsilon_s$, the maximum sensitivity of the proposed optical waveguide sensor is reached at the optimum thickness of the guiding layer. Thus the maximum sensitivity is defined when the derivative of $S$ with respect to $t$ (the film width) vanishes [17, 18] or $\frac{\partial S}{\partial t} = 0$.

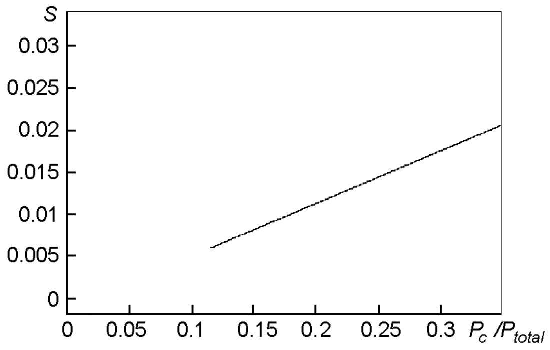

Fig. 1. Sensitivity of the proposed optical waveguide sensor versus the fraction of total power flowing in the cladding.

## 3. Results

To calculate the sensitivity behavior for the proposed sensor in terms of its structure parameters, we solved Eq.(8) numerically for the effective RI ($N$). Then, we applied the result into Eq.(9) to calculate the sensitivity. For optimal sensor design, all necessary information can be found on the surface $r_s(a_s, a_c)$ and the above set of equations. This can be performed as follows. The materials to be used in the sensor production are determined according to the sensor application, temperature, mechanical stability, cost criteria and optical and chemical stability. Then, the opto-geometrical parameters $a_s$ and $a_c$ are calculated from the permittivities of these materials. The value of the effective refractive index $N$ for the optimal sensor is calculated from the corresponding value of $r_s$. Then, the value of $t$ at which the sensor is characterized by maximum sensitivity can be evaluated from Eq.(8). The maximum sensitivity can be calculated by substituting the parameters of the optimal sensors which evaluated as described into Eq.(9).

Fig. 1 exhibits the relation between fraction of total power propagating in the cover medium ($P_c/P_{total}$) and sensitivity of the sensor. In the most cases, they are treated equally. Thus, the enhancement of the fraction of power flowing in the clad is essential for sensing applications.

Fig. 2 shows the sensitivity of the proposed sensor versus the LHM parameter of $|\mu_f/\mu_0|$. From Fig. 3, we notice that for small values of $|\mu_f/\mu_0|$, the sensitivity approaches zero. This is due to the high confinement of

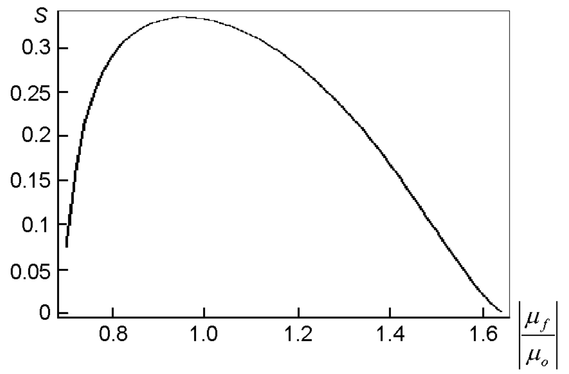

Fig. 2. Sensitivity of the proposed sensor versus the absolute value of $n$ where $\mu_f = n\mu_0$ for $a_c = -0.65$ and $a_s = -0.67$.

the guided mode in the guiding film. Far from this point at the other end of the curve, the sensitivity approaches zero again because this region corresponds to the cutoff at which total power of the mode propagated in the substrate and the sensing evanescent field in the cladding vanishes. In this case, the field does not sense variations in the RI of the cladding. Between these two limits, there is a maximum at which a relatively large part of the total power propagates in the cladding.

In Fig. 3 the variation of the sensitivity of the proposed sensor with the clad-film interface nonlinearity is illustrated. The sensitivity increases with increasing the clad-film interface nonlinearity. In our analysis we consider the nonlinear coefficient α to be positive. Thus increasing α will increase the RI of the nonlinear cladding. As the ratio of the RI of the cladding to the RI of the guiding layer ($n_c/n_f$) increases the confinement of the wave gets smaller and the penetration of the evanescent field into the cladding become larger. Therefore, the sensitivity of the senor increases.

In Fig. 4 and Fig. 5 the sensitivity is plotted versus the asymmetry parameters $a_s$ and $a_c$. It is clear that the sensitivity decreases with increasing $a_s$ and with decreasing $a_c$. As $a_s$ increases the guided field shifts towards the substrate and therefore the fraction of total power flowing in the substrate is enhanced reducing the evanescent field in the cladding and the fraction of total power flowing in the cladding. To obtain high sensitivity, it is essential to get as much of the optical power as possible to propagate in the nonlinear cladding medium. Thus decreasing $a_s$ and increasing $a_c$ will enhance the sensitivity of the proposed optical waveguide sensor.

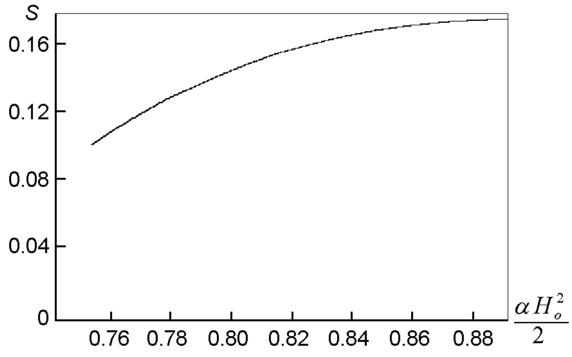

Fig. 3. Sensitivity versus the clad-film interface nonlinearity for $a_c = -0.6$, $a_s = -0.75$.

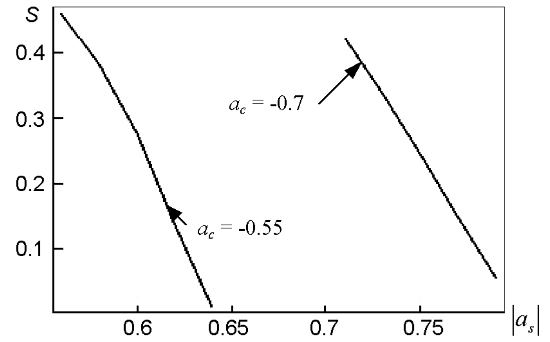

Fig. 4. Sensitivity as a function of the absolute value of the asymmetry parameter $|a_s|$ of the proposed sensor.

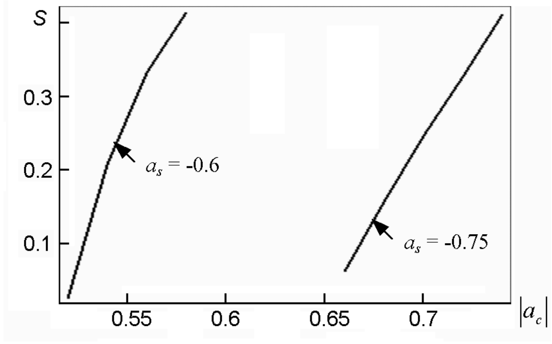

Fig. 5. Sensitivity as a function of the absolute value of the asymmetry parameter $|a_c|$ of the proposed sensor.

## 4. Conclusions

The optimal design for nonlinear-LHM-Linear slab waveguide structure sensor is studied as a function of the LHM parameters and the nonlinear coefficient. We evaluated theoretically the propagation of TM modes in the structure for homogeneous optical sensing applications. The LHM material is embedded between a semi-infinite linear substrate and a semi-infinite nonlinear cladding with an intensity-dependent refractive index of Kerr type. Results show that the sensitivity depends on the LHM parameters and the nonlinear parameters. The results are important for designer to consider the parameters for getting maximum sensitivity of future sensors such as the film thickness, the parameters of each layer of the sensor.

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# Нелінійний хвилепровідний пластинчатий оптичний сенсор на поперечно-магнітній моді з використанням лівоповоротних матеріалів

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Розглянуто пластинчату хвилепровідну структуру, яка має лівоповоротний матеріал (ЛПМ) в якості шару, що направляє хвилі. Теоретично досліджувалося розповсюдження поперечно-магнітних мод в структурі, що використовується для оптичного зондування однорідних середовищ. ЛПМ розташовано між напівнескінченною оптично лінійною підкладкою та напівнескінченним нелінійним покриттям з залежним від інтенсивності за законом Керра показником заломлення. Дослідження в основному спрямовано на визначення чутливості запропонованого оптичного сенсору для надання розробникам даних щодо оптимальної структури сенсора, що забезпечує максимальну чутливість.