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Stressed state of laminated interference-absorption filter under local loading

***L.Ya.Ropyak*[1], *M.V.Makoviichuk*[2], *I.P.Shatskyi*[2], *I.M.Pritula*[3], *L.O.Gryn*[3], *V.O.Belyakovskyi*[4]**

[1]Ivano-Frankivsk National Technical University of Oil and Gas, 15 Karpatska Str., 76019 Ivano-Frankivsk, Ukraine
[2]Ivano-Frankivsk Branch of Pidstryhach-Institute for Applied Problems in Mechanics and Mathematics, National Academy of Sciences of Ukraine, 3 Mykytynetska Str., 76002 Ivano-Frankivsk, Ukraine
[3]Institute for Single Crystals, STC "Institute for Single Crystals", 60 Nauky Ave., 61072 Kharkiv, Ukraine
[4]Private company "Modern Optical Technology", 16 Obolonskyi Ave., 04205 Kyiv, Ukraine

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The article describes a design of an interference-absorption short-wave cut-off filter which provides the minimum light transmission in the visible region of the spectrum from 0.65 to 0.78 μm and the maximum light transmission in the range 2.0 μm to 5.0 μm. An engineering technique for calculation of stressed state and strength of a layered light filter element under local loading has been developed. Acceptable loadings for a layered Si–SiO composition on sapphire substrate has been estimated.

Keywords: interference-absorption filter, layered coating, local loading, stresses, strength.

Напружений стан шаруватого інтерференційно-абсорбційного фільтру під локальним навантаженням. *Л.Я.Роп'як, М.В.Маковійчук, І.П.Шацький, І.М.Притула, Л.О.Гринь, В.О.Беляковський.*

Описано конструкцію інтерференційно-абсорбційного короткохвильового відсікаючого фільтра, який забезпечує мінімальне пропускання світла у видимій області спектра від 0.65 до 0.78 μm і максимальне у ближній інфрачервоній області від 2 до 5 μm. Розроблено інженерну методику розрахунку напруженого стану та міцності багатошарового елемента світлофільтра під дією локального механічного навантаження. Проведено оцінку допустимих навантажень для шаруватої Si–SiO композиції на сапфіровій підкладці.

Описана конструкция интерференционно-абсорбционного коротковолнового отсекающего фильтра, который обеспечивает минимальное пропускание света в видимой области спектра от 0.65 до 0.78 μm и максимальное — в ближней инфракрасной области от 2 до 5 μm. Разработана инженерная методика расчета напряженного состояния и прочности многослойного элемента светофильтра под действием локальной механической нагрузки. Выполнена оценка допустимых нагрузок для слоистой Si–SiO композиции на сапфировой подложке.

1. Introduction

Interference-absorption short-wave cut-off filters are used as optical elements for the range 0.78 μm–5.0 μm and can be used in optics and spectral studies. In addition, the aforementioned filters can be also used as a component of high-power optical devices to operate in the given spectral range. In particular, interference-absorbing short-wave cut-off filters serve as output windows of light-emitting devices. Thereat, the filters adjust the emission spectra of broadband emitters by transmitting light in the range 0.78 μm–5.0 μm and absorbing (reflecting) visible emission.

The interference-absorption short-wave cut-off filter developed by the authors of the study [1] (Fig. 1) consists a single-crystalline sapphire substrate coated with a transitional SiO layer. This layer, in its turn, is covered with the required number of alternating Si and SiO layers with $\lambda/4$ optical thickness, and the upper SiO layer is the thickest. The transparent substrate is made of single-crystal optical sapphire with a surface roughness *Ra* ranges from 8 to 12 nm. The thicknesses of the first and last SiO layers ranges from 10 to 15 nm and from 350 to 400 nm, respectively; Si film layers have a high refractive index. As is known, the reflection coefficient increases with the increasing number of layers, and the odd number of layers has a greater effect on the reflectivity of the interference coating than an even number. Therefore, the optimum number of the Si/SiO interference coating layers which meets the requirements to the integral transmission coefficient in the working long-wavelength spectral region ranges between 11 and 13.

The developed design of the interference-absorption short-wave cut-off filter provides the integral transmission coefficient in the visible spectral region (from 0.65 to 0.78 μm) less than 2 %, the integral transmission in the spectral region raged from 2 to 5 μm on the level of 80 % and more, as well as long lifetime in extreme conditions. Moreover, due to high thermal conductivity of sapphire, the filter for the infrared (IR) spectrum region can quickly remove heat from the heating zone, and makes the height of secondary maxima (higher-order harmonics) equal in the working IR region of the spectrum with simultaneous increase of the integral transmission coefficient in this region. It also diminishes the integral transmission coefficient in the visible region of the spectrum.

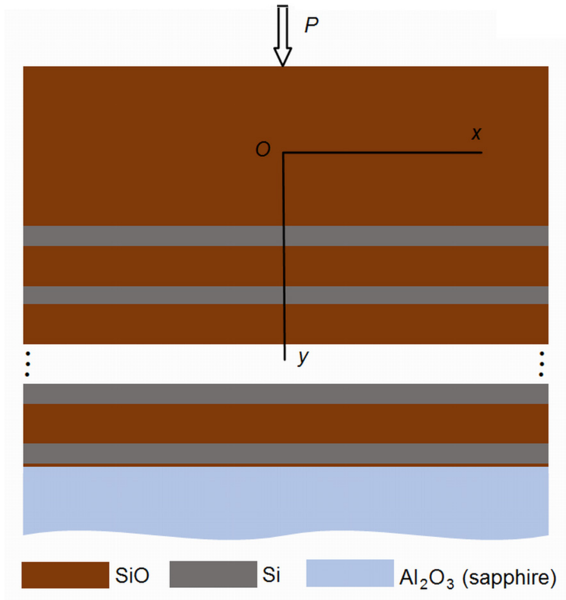

Fig. 1. A fragment of the laminated filter under local loading.

Cut-off filtering coatings are designed using special programs. However, mathematical modelling cannot completely meet all the required options of the coating. Therefore, it is supplemented with experimental data in the region of matching layers of the interference coating, which contacts with a substrate and air. The specified options of these layers enhance adhesion of the coating to the substrate, raise the value of optical transmission and smooth the secondary maxima in the considered spectral region [2, 3]. Authors mostly focus on crystallization processes of materials for filters [4, 5] and on optical properties of filter coatings [6], but do not pay appropriate attention to physic-mechanical properties of multi-layer coatings used in extreme conditions including high temperatures and mechanical loadings. There are several studies of stress concentration in coated solids near crack-like damages of coating [7] and substrate [8, 9], as well as methods for solution of thermal conductivity problems [10–12] and thermoelasticity [13–15] for solids with multi-layer coatings.

The presented herein research aims at developing the engineering technique for calculation of stressed state and strength of the multi-layer film coating for interference-absorption filter under local mechanical loading. In order to obtaining results in analytical form, the authors developed here the method of one-dimensional analysis proposed earlier for the two-layer compositions [16–18].

2. Modelling

Let us consider the structure element of the interference-absorption filter (Fig. 1). It consists of the crystalline sapphire substrate and multi-layer film coating made of alternating Si and SiO layers with the thicknesses h_{Si} and h_{SiO}, respectively. Thereat, the lower (transitional) h_{tr}-thick layer and the upper (protective) h_c-thick layers are made of SiO.

From the mechanical viewpoint, we model upper protective SiO-layer as a bending plate. At the same time, the operational compositional N-layer Si–SiO set meets the Winkler's hypothesis on proportionality of stresses and elastic displacements. For simplicity, the sapphire substrate is assumed absolutely rigid, and the mechanical contact between the components on the layer interfaces is ideal. This composition is loaded with normal force P (N/m) evenly distributed along the line perpendicular to the plane of the figure. Moreover, we assume the flat deformation state (ε_z=0). It should be studied the stress distribution in the layer composition of the filter structure and specified the acceptable local loading.

Taking into account the mentioned above assumptions, the equilibrium equation for the coating on the elastic substrate has the following form [19]:

$$D\frac{d^4u_y}{dx^4} + k_y u_y = P\delta(x), \quad x \in (-\infty,\infty). \tag{1}$$

Here u_y is the component of elastic displacement vector of the middle plate surface; $\delta(x)$ is the Dirac function; $D = E_c h_c^3/(12(1 - \nu_c^2))$ is the bending rigidity; k_y is the coefficient of integral rigidity for piecewise uniform substrate; $E_c = E_{SiO}$, $\nu_c = \nu_{SiO}$ are the Young's modulus and the Poisson's ratio of the coating material.

The stresses and moments vanish at infinity:

$$D\frac{d^2u_y}{dx^2}(\pm\infty) = 0, \quad D\frac{d^3u_y}{dx^3}(\pm\infty) = 0, \tag{2}$$

Let us define the coefficient of substrate rigidity for the multi-layer depth-inhomogeneous filter as the value inversely proportional to the total compliance of series-connected layers:

$$k_y = \left[\int_{h_c/2}^{h_c/2+(N-1)(h_{Si}+h_{SiO}) + h_{Si} + h_{tr}} \frac{dy}{E(y)}\right]^{-1} =$$

$$= \frac{1}{N-1}\left(\frac{h_{Si}}{E_{Si}} + \frac{h_{SiO}}{E_{SiO}}\right) + \frac{h_{Si}}{E_{Si}} + \frac{h_{tr}}{E_{SiO}}.$$

Thus, the boundary problem (1), (2) describes the required field of vertical displacements of the coating-substrate/plate on the elastic layered substrate.

3. Results and discussion

The problem (1), (2) is solved in the following form

$$u_y(x) = \frac{P}{8D\lambda_y^3}e^{-\lambda_y|x|}(\cos\lambda_y x + \sin\lambda_y|x|), \tag{3}$$

where $\lambda_y = \left(k_y/(4D)\right)^{1/4}$ is the subgrade reaction ratio with the dimension inverse to the length.

The bending moment in the coating corresponds to displacements (3):

$$M(x) = \frac{P}{4\lambda_y}e^{-\lambda_y|x|}(\cos\lambda_y x - \sin\lambda_y|x|).$$

In particular, for the stresses at the bottom base of the coating ($y = h_c/2$) we obtain the relation

$$\sigma_x = \frac{6M}{h_c^2} = \frac{3P}{2h_c^2\lambda_y}e^{-\lambda_y|x|}(\cos\lambda_y x - \sin\lambda_y|x|). \tag{4}$$

The stresses in the filter layers should have the form

$$\sigma_y = -k_y u_y = -\frac{P}{2}\lambda_y e^{-\lambda_y|x|}(\cos\lambda_y x + \sin\lambda_y|x|). \tag{5}$$

The strength of each layer can be estimated in accordance with the von Mises criterion. Therefore, the strength condition for the plane deformed coating is as follows

$$\sigma_{eq} \equiv \left\{(1 - \nu_c + \nu_c^2)(\sigma_x^2 + \sigma_y^2) - \right. \tag{6}$$
$$\left. - (1 + 2\nu_c - 2\nu_c^2)\sigma_x\sigma_y\right\}^{1/2} \le [\sigma]_{SiO}$$

for the Winkler multi-layer it has the following form

$$\sigma_{eq} \equiv |\sigma_y| \le \min\{[\sigma]_{Si}, [\sigma]_{SiO}\}. \tag{7}$$

Here $[\sigma]_{Si}$, $[\sigma]_{SiO}$, are the admissible stresses for silicon and silicon oxide components of the filter.

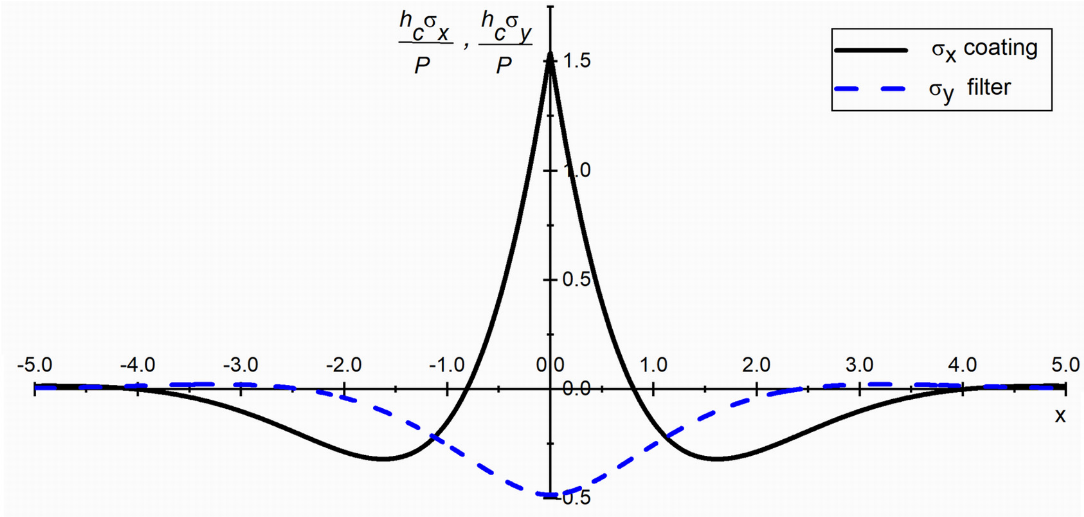

Fig. 2. Distribution of normal loadings σ_x in the coating at the boundary of contacts between the layers ($y = h_c/2$) and σ_y in laminated filter.

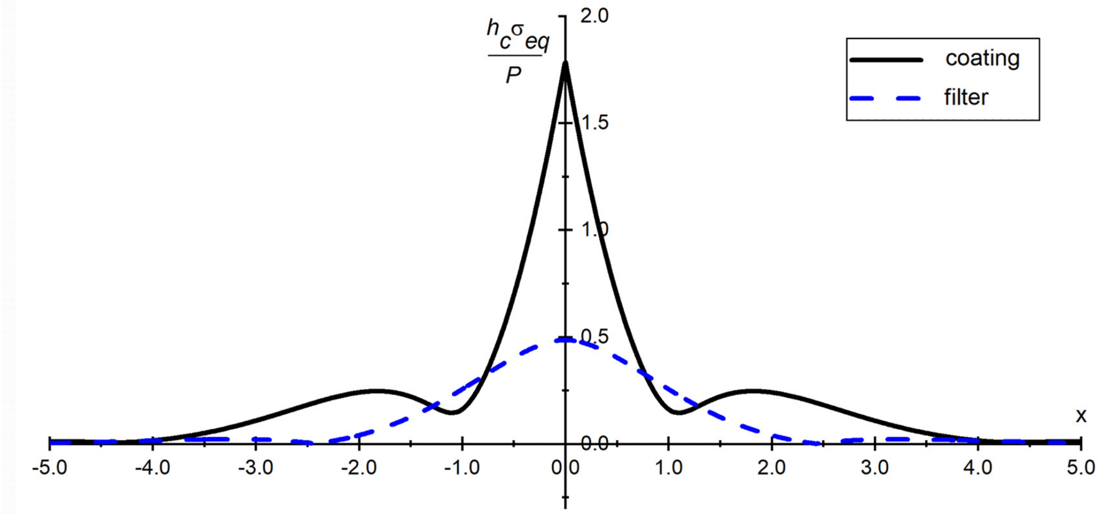

Fig. 3. Distribution of equivalent stresses in the coating at $y = h_c/2$ and in laminated filter.

The obtained analytical results (4)–(7) were analyzed for the composition containing the silicon oxide coating with the parameters: $h_c = 400$ nm, $\nu_c = \nu_{SiO} = 0.17$, $E_c = E_{SiO} = 73$ GPa, $[\sigma]_{SiO} = 110$ MPa; the silicon oxide transition layer with $h_{tr} = 10$ nm and the silicon layer with the parameters: $h_{Si} = 50$ nm, $E_{Si} = 131$ GPa, $\nu_{Si} = 0.266$, $[\sigma]_{Si} = 130$ MPa; the silicon oxide layer with the parameters: $h_{SiO} = 100$ nm, $\nu_{SiO} = 0.17$, $E_{SiO} = 73$ GPa, $[\sigma]_{SiO} = 110$ MPa. The number of bi-layers was $N = 11$.

Fig. 2 shows the stressed state of the composition. The most critical point is $x = 0$, $y = h/2$ for the considered case.

The values of admissible loadings P_* were found from the conditions (6), (7) taking into account, that $\max_x \sigma_{eq}(x) = \sigma_{eq}(0)$. In particular, as Fig. 3 shows, $\sigma_{eq} \approx 1.8P/h_c$ for the coating. Then the maximum permissible distributed load along the loading line should have the value

$$P_* = \frac{[\sigma]_{SiO} h_c}{1.8} =$$

$$= \frac{110 \cdot 10^6 \cdot 400 \cdot 10^{-9}}{1.8} = 24.44 N/m.$$

For the plate with the width $h = 78.8$ mm the total boundary force is $F_* = P_* b = 24.44 \cdot 78.8 \cdot 10^{-3} = 1.92 \approx 2$N.

This value can be essentially increased taking a value close to the theoretical strength of silicon oxide instead of $[\sigma]_{SiO} = 110$ MPa for the nano-layer.

4. Conclusions

The developed the calculation technique makes possible to determine analytically the influence of the relation between geometric and mechanical characteristics of nano-layers on the stressed state and the boundary equilibrium of the interference-absorption filter.

The principal point in the proposed technique is the use of strength criteria for all components of the piece-homogeneous layered coating.

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