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Formation of "windows" of transparency in a layered sample with an inhomogeneous distribution of inclusions in the layers. General case

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In this work, we studied the formation of transparency "windows" in a layered sample, the layers of which consist of an opaque material with transparent inclusions. The case when islands of a transparent material are distributed inhomogeneously in a layer is considered. The distribution densities of the "windows" over the area were obtained for inhomogeneities of various types and different values of the "thickening" of the islands to the center or edge of the layer. The general case of many layers and islands in a layer is investigated.
Keywords: "windows" of transparency, inhomogeneous distribution of islands, central and edge inhomogeneity.

Формування "вікон" прозорості у шаруватому зразку при неоднорідному розподілі включень у шарах. Загальний випадок. *Р.Є.Бродський*

Вивчено формування "вікон" прозорості у шаруватому зразку, шари якого складаються з непрозорого матеріалу з включеннями прозорого. Розглянуто випадок, коли острівці прозорого матеріалу розподілені неоднорідно у шарі. Отримані густини розподілу "вікон" за площею для неоднорідності різного виду і різної величини "згущення" острівців до центру або краю шару. Досліджено загальний випадок багатьох шарів і острівців у шарі.

Изучено формирование "окон" прозрачности в слоистом образце, слои которого состоят из непрозрачного материала с включениями прозрачного. Рассмотрен случай, когда островки прозрачного материала распределены неоднородно в слое. Получены плотности распределения "окон" по площади для неоднородности различного вида и различной величины "сгущения" островков к центру или краю слоя. Исследован общий случай многих слоёв и островков в слое.

1. Introduction

In a layered sample, individual layers of which may contain, in addition to the base material, inclusions of additional material with properties different from the base one, bulk inclusions, including those penetrating through the sample; may appear, when inclusions in adjacent layers can contact each other. Such bulk inclusions can qualitatively change the properties of the sample. An example is transparent inclusions in an opaque material. If the transparent inclusions are located strictly one above the other in all layers of the sample, a "window" of through transparency will appear in the sample. The formation of the transparency "windows" in the sample depends on the number of layers, the number and size of the islands of inclusions in the layers, and on the type of distribution of these islands in the layer.

The goal, set in the study of the formation of transparency "windows" is to obtain the distribution of "windows" over the area for the given parameters of the formation of inclusions in separate layers.

The case of a uniform distribution of islands in a layer that is the same in all layers was considered in [1]. The cases, when the formation of islands in a given layer depends on their location in the previous layer, are considered in [2–4].

In this paper, we consider the case of the formation of islands, independent and identical in all layers and inhomogeneously distributed over the layer. The probability of nucleation of an island (i.e. the probability of the location of the center of the island) in a given element of the layer area depends on the position (coordinates) of the element in the layer. In [5], the results of a study of the formation of transparency "windows" during the nucleation of islands inhomogeneously distributed over a layer are presented for a particular case — the case of two layers with a single round transparent island in each; the case has special solutions, and in some cases allows an analytical solution.

In this paper we present the results of studying the case of many layers and many islands. Even if transparent inclusions consist of simple islands, for example, round, their overlap inside the layer and intersection with the formation of a "window" leads to the fact that the formed "window" can have the very diverse area and complex shape; therefore, the study of this case will be carried out using a numerical modeling.

The purpose of this work is to study the influence of distribution inhomogeneity, different types and sizes +, on the formation of transparency "windows" in the general case of many layers and many islands in a layer.

2. Problem state

We will consider a cylindrical sample consisting of $N \geq 2$ round layers of radius R. Each layer consists of a basic opaque material and transparent inclusions. The inclusions are many round islands, possibly overlapping, with a radius r. The number of islands in a layer is n. The layers are considered "thin", their thickness is not taken into account and does not affect the formation of "windows".

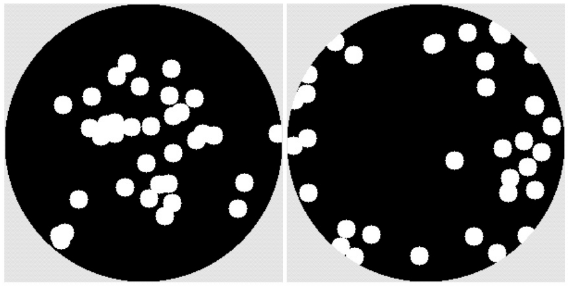


Fig. 1. Layer with inclusions; the case of "central" (left) and "edge" (right) inhomogeneity of the distribution of islands (opaque material is shown in black, transparent — in white, background — in light gray).

In this paper, the case of an inhomogeneous distribution of islands in a layer is considered; the change in the distribution of "windows" over the area is studied when changing, first of all, the type and degree of inhomogeneity, as well as the number of layers and islands in the layer.

The type of distribution of islands in the layer, as in [5], is specified by the density $f(x,y)$ of distribution of their centers over the area of the layer. As in [5], we will consider centrally symmetric distributions. Let's introduce ρ *a* distance from the given element of the layer to the center of the layer, divided to the radius of the layer R. Let's introduce the density $f(\rho)$ of distribution of islands such that $f(\rho) \cdot d\rho \cdot \rho d\varphi$ is the probability of nucleation of an island in an element $(\rho, \rho + d\rho)\times(\varphi, \varphi + d\varphi)$ of the layer, $f(\rho)$ does not depend on φ. We assume that in each layer, n islets are distributed, each with a density $f(\rho)$. The function $f(\rho)$ is normalized to one. With a uniform distribution of islands: $f(\rho) = f_0 = 1/\pi$.

A decrease in $f(\rho)$ corresponds to a higher probability of the nucleation of islands at the center of the layer, we will call such $f(\rho)$ "central"; an increase in $f(\rho)$ corresponds to a higher probability of nucleation of islands at the edge of the layer, such $f(\rho)$ we will call "edge". Fig. 1 shows a typical view of a layer with inclusions in the case of "central" (left) and "edge" (right) density function $f(\rho)$.

To study the entire range of variation in the degree of inhomogeneity, we will use the same density functions $f(\rho)$, as in [5] — Gaussian $f(\rho) = A\exp(-\alpha\rho^2)$ and power $f(\rho) = A\rho^{\alpha}$. Coefficients A, α in the functions are related by the normalization condition, so that the functions have one control parameter. When $\alpha = 0$, the homogeneous case is realized; with increasing α one can get an arbitrarily sharp probability "peak" in the center of the layer in the case of a Gaussian function, and an arbitrarily narrow "ring" of the probable position of the centers of the islands at the edge of the layer for a power function.

For investigating small values of inhomogeneity, as well as for comparing the distributions of "windows" at the same degrees of inhomogeneity, but different $f(\rho)$, we will also use, as suggested in [5], the linear $f(\rho) = A \pm \alpha\rho$, central ("−") and edge ("+"), as well as the quadratic, parabolic $f(\rho) = A \pm \alpha\rho^2$ density function.

As a numerical characteristic of the "degree of inhomogeneity" we will use the distance ρ_w to the center of the layer, such that the probability of nucleation of an island "inside" it ($\rho<\rho_w$) for a given $f(\rho)$ is equal to 0.5 (and "outside" it is the same). The stronger the "central" inhomogeneity, the less ρ_w; the stronger the "edge" inhomogeneity, the closer ρ_w to one. The value ρ_w for the homogeneous case we denote $\rho_{w0} = 1/\sqrt{2}$ ([5]).

Note that possible ρ_w values for linear and quadratic $f(\rho)$ functions are limited because the possible values of function parameters are limited by the condition $f(\rho)$ at $\rho\in[0,1]$, which was performed automatically for the Gaussian and power functions.

For "central" ("−") linear $f(\rho)$ function, this condition is important at the edge, $f(1)\geq 0$, whence the minimum possible value ρ_w for a "central" linear function $\rho_w = 0.5$ (from the condition $f(1) = 0$, taking into account the normalization [5]). Similarly, for "edge" ("+") linear $f(\rho)$ function, this condition has a meaning in the center, $f(0)\geq 0$, and the maximum possible value ρ_w for a "edge" linear function $\rho_w = \frac{1}{2^{1/3}} \approx 0.7937$.

Similarly, for parabolic (central, "−") functions, the minimum $\rho_w = \sqrt{1 - \frac{1}{\sqrt{2}}} \approx 0.54$, and the maximum (for edge, "+") $\rho_w = \frac{1}{2^{1/4}} = 0.841$.

The central linear $f(\rho)$ function pulls the islands to the center more strongly than the parabolic one, but the edge parabolic one pulls the islands closer to the edge more strongly than the linear one.

As shown above, the minimum ρ_w (the strongest inhomogeneity) for the "central" linear inhomogeneity is 0.5. We will use the Gaussian $f(\rho)$ for ρ_w equal to this value and less. Namely, for $\rho_w = 0.1$, $\rho_w = 0.3$, and $\rho_w = 0.5$.

For a power function ("edge" case), we use ρ_w such that they split the interval (1, ρ_{w0}) (from largest to smallest) in the same proportion as those selected above for the Gaussian function ρ_w split interval (0, ρ_{w0}) corresponding to the "central" inhomogeneity.

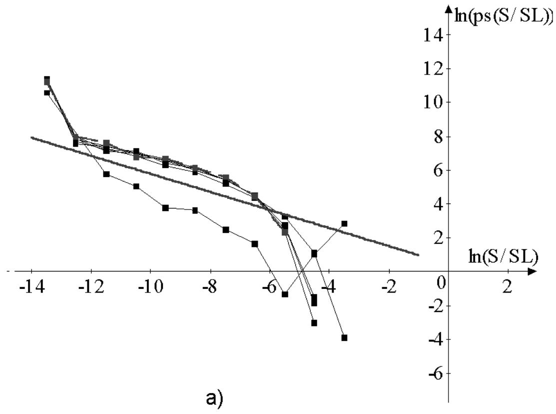
Value $\rho_w = 0.1$ will match the value $\rho_w \approx 0.96$, value $\rho_w = 0.3$, will match $\rho_w \approx 0.876$, and value $\rho_w = 0.5$ will match $\rho_w \approx 0.7929$. The last value practically coincides with the maximum ρ_w for the linear $f(\rho)$ function, which will allow us to compare the results for linear and power $f(\rho)$.

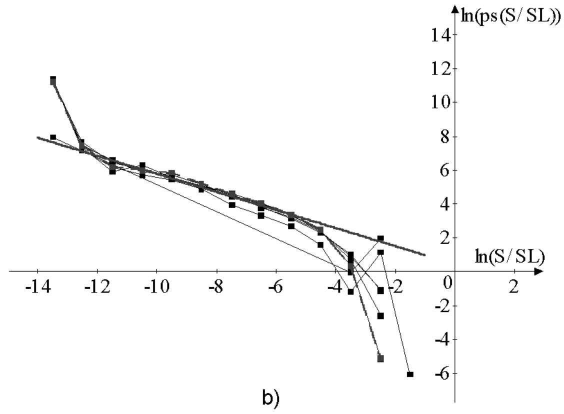
3. Distribution of "windows" of transparency; general case

In general, the number of islands n and the number of layers N can be arbitrary. In [1], for an arbitrary N and one island in a layer with a homogeneous island distribution, a small-scale asymptotics of the distribution density of "windows" over the area was found analytically (approximately); this asymptotics had the form $p_{s0}(S) = 1/\sqrt{S}$. The results of a numerical experiment for $n = 5...20$ and a small N ($N = 3$ and $N = 4$) have shown that in this case, the density has a small-scale asymptotics $p_s(S) \approx S^{\alpha}$ at $\alpha \approx -0.5$, i.e. this kind of density is fairly stable.

Further in this section, the results (Fig. 2–5) for the general case with a non-uniform distribution of islands in the layer are presented.

The cases selected for simulation were $N = 3$ (which was considered in [1] for the homogeneous case) and $N = 10$. With such a change in N, the influence of the number of layers on the distribution density of "windows" is already becoming visible, but we can still use the number of islands n and





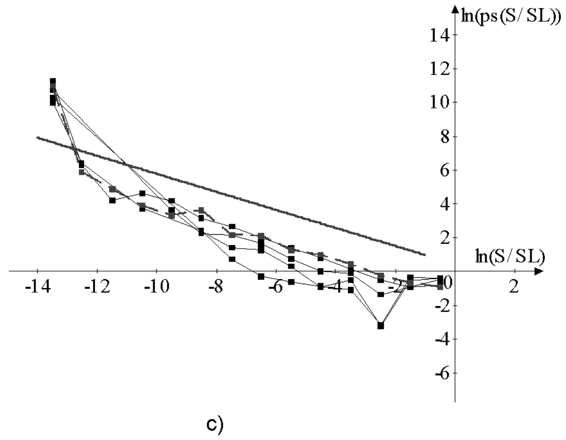


Fig. 2. $N = 3$ layers, "central" inhomogeneity: a) – r=0.1R; b) – r=0.2R; c) – r=0.4R.

the radii of the islands r, the same as for the case $N = 3$, which is important for the comparison of results. The number of islands was taken $n = 15$ in all the experiments. The radii of the islands at which, with given N, n, non-trivial results are obtained (not only almost opaque and not only almost completely transparent samples) are in the range $r = 0.1\ldots0.6R$.

Of course, we leave aside a very wide range of changes in the parameters, but the purpose of this work is to study the influence of the inhomogeneity of the distribution of islands in the layer on the formation of "windows". We will leave a more detailed study of the influence of other parameters for future work.

Figs. 2–5 show the plots $p_s(S)$ of distribution densities of "windows" over area for the selected parameters and the types of inhomogeneity of the distribution of islands listed below; the graphs are given in a double logarithmic scale.

It can be seen that in all Figs. 2–5, the general view of the graphs is similar: on the left you can see a rapidly decreasing section, then a section of a slower decrease and again a rapidly decreasing large-scale section. The very first section mentioned corresponds to the "windows" with an area of several pixels in a numerical experiment; in this section the counting error is very large, we will not dwell on it. The second region thus corresponds to the small-scale part of the density $p_s(S)$ and the last one to the large-scale part. As mentioned above, in the homogeneous case, the small-scale part of the density is a power-law, i.e. linear on a double logarithmic scale with the slope of the linear approximation equal to the asymptotic exponent about -0.5.

In all Figs. 2–5, in addition to the densities for the cases of an inhomogeneous distribution of islands, graphs are shown for a uniform distribution for used N, n, r in gray, with dots.

In [1], the small-scale asymptotics — a linear approximation in the small-scale region of the graph in a double logarithmic scale — was found for the homogeneous case with $N = 3$, $r = 0.2R$. Here we also found a linear approximation for these parameters; it is shown by a gray solid line in all graphs in Fig. 2–5 to compare the results for different N, r.

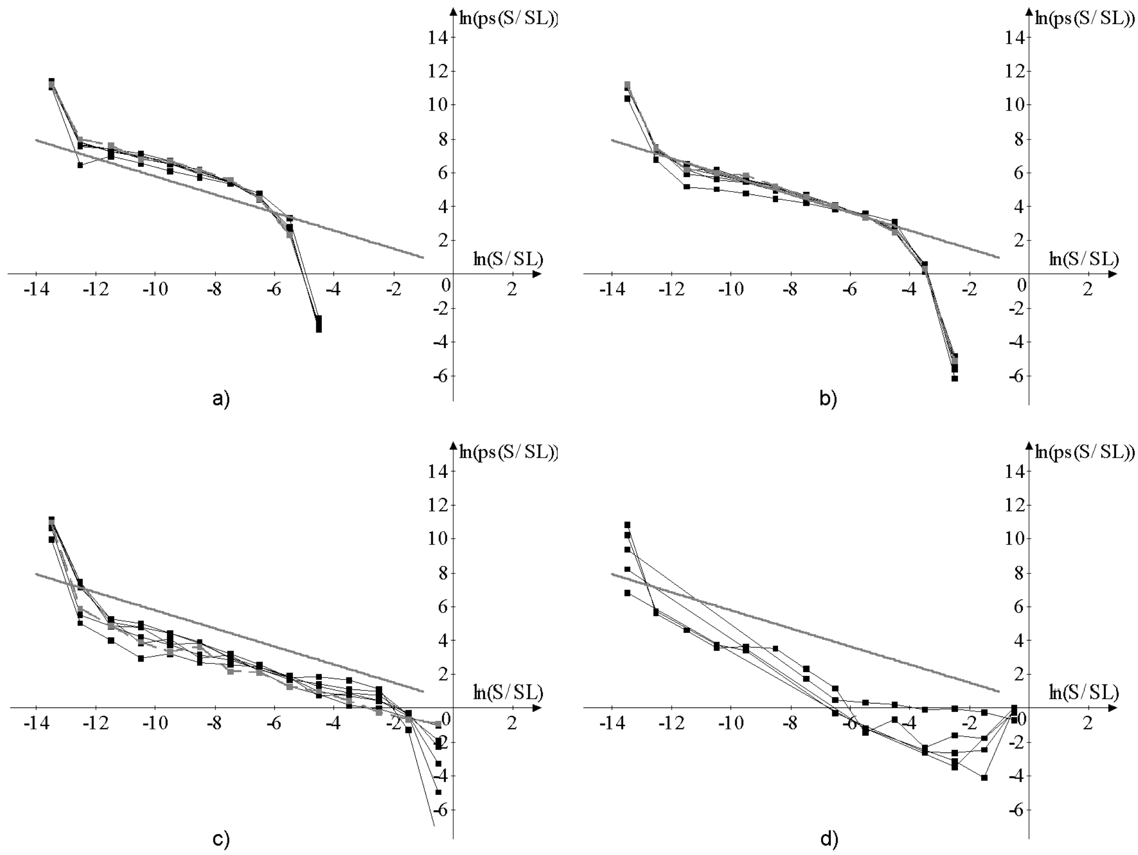


Fig. 3. N = 3 layers, "edge" inhomogeneity: a) – r=0.1R; b) – r=0.2R; c) – r=0.4R; d) – r=0.6R.

When considering the case of two layers in [5], it was shown that small changes in the inhomogeneity have little effect on the distribution of the formed "windows"; therefore, here, in order not to bring a large number of very close graphs, we studied only the linear and Gaussian "central" inhomogeneity and only the parabolic and power-law "edge" (linear "central" is stronger than parabolic, parabolic "edge" is stronger than linear, as shown above).

Fig. 2 shows the graphs $p_s(S)$ of distribution densities of "windows" by area for inhomogeneity of "central" type, linear and Gaussian, for N = 3 and $r = 0.1R$ (Fig. 2a), $r = 0.2R$ (Fig. 2b), $r = 0.4R$ (Fig. 2c). Thus, in each of Fig. 2a, 2b and 2c, graphs for all types of "central" inhomogeneity at the same value r are shown.

When $r = 0.6R$ (the largest of the simulated values), all formed "windows" (besides the smallest ones mentioned above, consisting of several pixels) are so large that instead of a set of points in the experiment, one or two points are obtained in a large-scale region, therefore we will not consider the value $r = 0.6R$ for $N = 3$ in the case of a "central" inhomogeneity.

Let's start discussing the results from the case $r = 0.2R$; for comparison with the homogeneous case [1], we constructed a linear approximation of the small-scale region, with which we will further compare the remaining plots. It can be seen that most of the graphs practically coincide.

Only graphs for the largest values of inhomogeneity are selected — for Gaussian inhomogeneity at $\rho_w = 0.3$ (passes just below the others, contains all points) and for $\rho_w = 0.1$. (The last "graph" contains only three points connected by lines for convenience, and further, with the exception of Fig. 3d, such graphs will not be shown. Instead, it will be indicated to what values of inhomogeneity it is possible to obtain graphs with a sufficiently large number of points). In the latter case, the above occurs — all the islands are so close to the center that only large "windows" are formed, there are simply no "windows" in most of the considered range. For the same reason, in the graph for $\rho_w = 0.3$, an increasing sec-

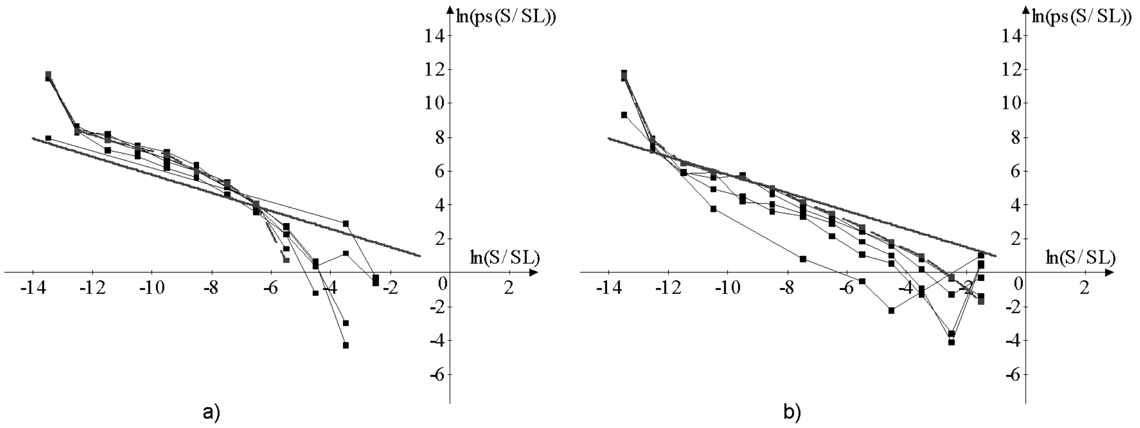


Fig. 4. $N = 10$ layers, "central" inhomogeneity: a) – r=0.2R; b) – r=0.4R.

tion appears in a large-scale region, although "windows" with other areas are still formed here. And for the same reason, the graph for $\rho_w = 0.3$ passes below the majority, this means a decrease in the total number of "windows" in the small-scale region at the expense of the large-scale one.

The angular coefficient of linear approximation of the small-scale region (for the homogeneous case shown by the gray line) practically the same in all graphs, except for $\rho_w = 0.3$, $\rho_w = 0.1$. This slope, i.e. the exponent of a small-scale power-law asymptotics of $p_s(S)$, is $\gamma \approx -0.5356$. The graph for $\rho_w = 0.3$ is more inclined, i.e. the index γ has a larger absolute value (smaller value, since it is negative). This coefficient increases in magnitude (the slope of the small-scale region increases) with an increase in the degree of inhomogeneity for other plots, but it is difficult to notice visually.

Let's turn to the case of "central" inhomogeneity, $r = 0.1R$ (Fig. 2a). As can be seen, for this case, the general regularities are the same as for the case above. Most of the plots coincide, the plot for the greatest inhomogeneity goes below the rest and contains an increasing section in the large-scale region (here at smaller $r = 0.1R$, this is $\rho_w = 0.1$, it is possible for it to obtain all points, both in small- and large-scale regions). The slope of the linear approximation for it in the small-scale region is also greater than for the others.

Let us now compare the results for this case with the results for $r = 0.2R$. The gray line in Fig. 2a is the linear approximation from the previous case (2b). It can be seen that the graphs for $r = 0.1R$ pass above this approximation in the small-scale region, i.e. the proportion of small "windows" is higher here, which is to be expected. Here, the slope of the linear approximation of the small-scale region is slightly less in absolute value ($\gamma \approx -0.505$) than in the case above. The slope for the strongest inhomogeneity is noticeably larger, $\gamma \approx -0.82$.

Fig. 2c shows the plots for the same "central" inhomogeneity at $r = 0.4R$. The general appearance of the graphs and the change with increasing inhomogeneity are the same, but the shift of the graphs with increasing inhomogeneity is much more noticeable than in the previous cases. With an increase in inhomogeneity, the graph in the small-scale region shifts downward, the slope increases.

The largest inhomogeneity for which the graphs are shown in this case is Gaussian at $\rho_w = 0.5$. For smaller r (Fig. 2a, 2b), it was possible to obtain graphs for $\rho_w = 0.3$ and $\rho_w = 0.1$ corresponding to a stronger inhomogeneity; but here, for such ρ_w, only two or three points are obtained in the large-scale region. In all the plots obtained, even with such a large island radius, an increasing section appears in the large-scale region.

Let's move on to the results obtained for the "edge" inhomogeneity. Fig. 3 shows the graphs $p_s(S)$ for the case of "edge" inhomogeneity for $N = 3$ layers. In all the graphs, the solid gray line shows the mentioned above linear approximation in the small-scale region for the homogeneous case at $r = 0.2R$.

As you can see, the graphs for $r = 0.1R$ (Fig. 3a) are shifted upward relative to the straight line in the small-scale region, as well as in the case of the "central" inhomogeneity (there one smaller "windows"). For the graphs of the "edge" inhomogeneity, the difference is that the slope of the linear small-scale region decreases with an increase in the inhomogeneity; this is clearly seen for the graph with the strongest inhomogeneity (in Fig. 3a, this is a power-law inhomogeneity at $\rho_w \approx 0.96$). The exponent for the small-scale asymptotics of this graph is $\gamma \approx -0.4297$, for others $\gamma \approx -0.4318$, i.e. the difference is small.

Fig. 3b shows graphs for $r = 0.2R$. As it is seen, most of the plots here, as in the case of the "central" inhomogeneity, are close to each other and to the homogeneous case (shown in gray). As with $r = 0.1R$, the main difference from the case of a "central" inhomogeneity is that with an increase in the inhomogeneity, the slope of the graphs decreases in the small-scale region (γ decreases in absolute value). This is most noticeable in the graph for the greatest inhomogeneity (here, a power-law inhomogeneity at $\rho_w \approx 0.96$ is also observed). For this graph, $\gamma \approx -0.29$, the rest of the graphs are close to the homogeneous case, for them $\gamma \approx -0.53$. The changes are stronger than for $r = 0.1R$.

For the "edge" inhomogeneity, it is also possible to obtain graphs for $r = 0.4R$ and for $r = 0.6R$. On the graphs for $r = 0.4R$ (Fig. 3c), the changes in the slope of the graph with a change in the inhomogeneity are clearly visible not only for the strongest inhomogeneity, but also for its smaller values (the graphs intersect in the region $\ln(S/S_L) \approx -6$.

Note that there is no increasing section in the large-scale region in Fig. 3a–c; because of a number of large "windows" at the "edge", "pulling" the islands to the edge of the layer, inhomogeneity is not formed here.

Finally, Fig. 3d shows the simulation results for $r = 0.6R$. An increasing section already appears in the large-scale region (the effect of the island size becomes stronger than the effect of the "edge" inhomogeneity). And the number of points in the small-scale region, for which it was possible to obtain nonzero $p_s(S)$ values in the experiment, decreases. Graphs can be obtained only for relatively large values of inhomogeneity — for the maximum parabolic ($h = 0$) and for the power-law. And for these graphs, the number of "windows" obtained in the experiment in the small-scale region is small, so the spread of values is large. However, a decrease in the slope of the graph with an increase in inhomogeneity can be seen here, at least in the example of the graph for the maximum inhomogeneity ($\rho_w \approx 0.96$), which is in the interval $(-6, -2)$ passes near the horizontal axis. In this region, $\gamma \approx -0.1378$.

Now we consider the simulation results for $N = 10$ layers. As stated above, studying of the change of $p_s(S)$ with changing N is not the main goal of this work; this work is devoted to the study of the influence of the inhomogeneity of the island distribution in the layer. However, we have performed simulations for different N in order to find out, how the influence of inhomogeneity changes with N.

Fig. 4 shows graphs $p_s(S)$ for the case of a "central" inhomogeneity for $N = 10$.

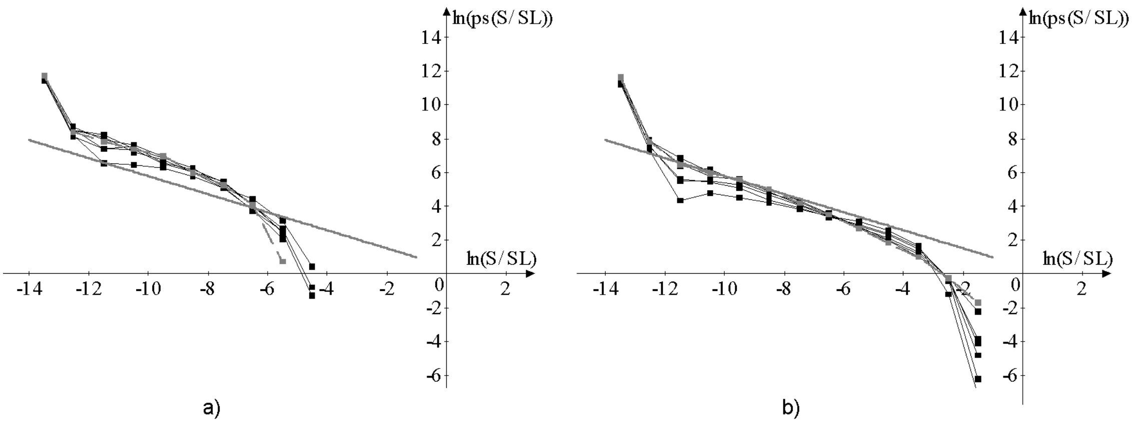
With such an increase in the number of layers, no noticeable number of "windows" were formed for the island radius $r = 0.1R$; the minimum radius from the simulated ones, for which it was possible to obtain graphs is $r = 0.2R$ (Fig. 4a).

Fig. 4a shows that an increase in the number of layers affects the appearance of $p_s(S)$ dependence; this effect is to some extent similar to that with a decrease in the radius of the island with the same number of layers. The graphs in Fig. 4a ($N = 10$, $r = 0.2R$) are shifted relative to the solid gray line (the same as above) similar to the graphs in Fig. 2a ($N = 3$, $r = 0.1R$). The graphs were constructed for all simulated values of the "central" inhomogeneity (linear and Gaussian, three values of the control parameter). But the graph for the maximum inhomogeneity here contains only 3 points — almost only large "windows" are formed, there are no points in the small-scale region.

The rest of the graphs in the small-scale region are shifted upward relative to the gray line, that is, there are more small "windows" than there were at $N = 3$, there are fewer large ones. The slope of the graphs, as before, increases with an increase in the "central" inhomogeneity.

When $r = 0.4R$ (Fig. 4b), the change in the slope of the graph and its downward shift in the main part with a change in the magnitude of the inhomogeneity are clearly noticeable, as in the case of $r = 0.4R$ for $N = 3$ (Fig. 2c). An increasing section also appears in the large-scale region; it is associated with the formation of a large number of large "windows" at such a large value of the island radius.

Finally, let us turn to the case of an "edge" inhomogeneity at $N = 10$ (Fig. 5).



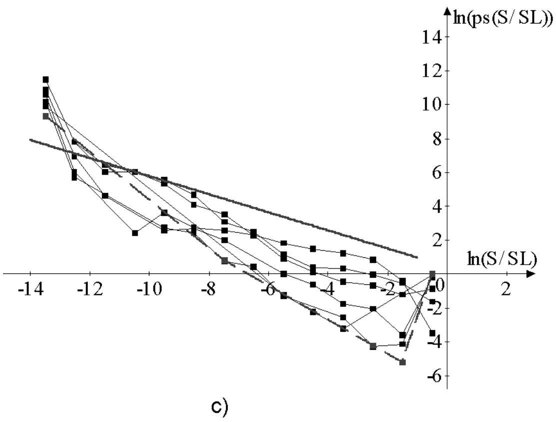


Fig.5. N=10 layers, "edge" inhomogeneity: a) – r=0.2R; b) – r=0.4R; c) – r=0.6R.

The general changes in the graphs relative to the case $N = 3$ are the same as for the "central" inhomogeneity. And as in the case of $N = 3$, the most noticeable difference from the case of a "central" inhomogeneity is that with an increase in the inhomogeneity, the slope of the graphs at the "edge" inhomogeneity decreases (γ decremented in absolute value).

The graphs for $r = 0.2R$ are shifted upward relative to the gray line — approximations for the homogeneous case $N = 3$, $r = 0.2R$ — in the small-scale area. The plot for the greatest inhomogeneity (power-law at $\rho_w \approx 0.96$) has a noticeably smaller slope in the small-scale region.

The graphs for $r = 0.4R$, in contrast to the case of the "central" inhomogeneity, pass quite close to each other. Those of them that correspond to the maximum inhomogeneity, i.e. with the smallest slope, pass close enough to the gray line, i.e. to the homogeneous case for $N = 3$, $r = 0.2R$ ($\gamma \approx 0.53$).

In general, the plots with "edge" inhomogeneity can be obtained for a wider range of values r, the radii of the islands, and these graphs have a smaller scatter at the same r, than in the case of "central" inhomogeneity.

The data for the graphs at $r = 0.6R$ (Fig. 5c) can be obtained in a fairly wide range of "window" areas, in contrast to the case of "central" inhomogeneity. A decrease in the slope of the graphs with an increase in inhomogeneity is clearly visible in these graphs (the graph for the homogeneous case, shown in gray, has the greatest slope). Here, as in the cases above, at a sufficiently increasing r, an increasing section of the large-scale region becomes noticeable.

4. Conclusions

The work investigates the formation of transparency "windows" in a layered sample, each layer of which consists of the main opaque material and inclusions of transparent islands. In the cases studied, the islands are distributed in a layer either with a "central" inhomogeneity, that is an increased probability of nucleation of islands closer to the center in comparison with the homogeneous case, or with an "edge" inhomogeneity with an reverse bias probability of island nucleation. The inhomogeneity was considered to be centrally symmetric. The degrees of inhomogeneity were studied in a wide range of inhomogeneity values ρ_w from 0.1 to 0.96.

For several types of inhomogeneous distribution of islands in a layer in a given range ρ_w, distribution densities $p_s(S)$ by the area of "windows" formed in the sample were obtained.

This paper presents the results of a study of the general case of an arbitrary number of layers N and an arbitrary number of islands n; the study was carried out using a numerical simulation for $N = 3$ and $N = 10$ at $n = 15$ and $r = 0.1R...0.6R$ (parameters n, r were selected close to [1] for comparison with the homogeneous case). From the simulation results (Fig. 2–5) it follows that the general view of the graphs remains the same in a wide range of parameters. The graphs show a small-scale region close to linear decreasing (in double logarithmic scale), and a large-scale region of rapid decreasing, or having in the case of strong heterogeneity, an increase part. Thus, for all investigated types of inhomogeneity, $p_s(S)$ has a power-law small-scale asymptotics with a negative exponent as in the homogeneous case. For various types and values of inhomogeneity, the following results were obtained.

When $N = 3$ for the case of a "central" inhomogeneity, it was possible to obtain the values $p_s(S)$ in both small and large scale regions for $r = 0.1R$, $r = 0.2R$ and $r = 0.4R$. For large r ($r = 0.6R$), "windows" are generated only in the large-scale region and the plots for $r = 0.6R$ were not built. On the graphs for every r, an increase in the slope of the small-scale region with an increase in inhomogeneity was observed. This increase was visually noticeable for the largest considered inhomogeneity $\rho_w = 0.3$ and $\rho_w = 0.1$. Also, for large values of inhomogeneity, an increasing section appeared in the large-scale region on the graphs — the proportion of large "windows" sharply increased. With an increase in r, differences between the graphs with different values of inhomogeneity increase.

When $N = 3$ for the case of an "edge" inhomogeneity, we was able to obtain the values $p_s(S)$ as for $r = 0.1R...0.4R$ (as in the case above) as for $r = 0.6R$. In general, the "edge" inhomogeneity has a "stabilizing" effect — the differences in the graphs with different values of inhomogeneity at the same r are noticeably less than with the "central" inhomogeneity, and the graphs can be obtained for a larger range of r. With increasing inhomogeneity, for all r, a decrease in the slope of the small-scale region was observed in the graphs (the slope of the linear approximation γ decreased in absolute value). As above, for small r, the change in the slope is visually noticeable for the greatest inhomogeneity, and for large r, the change in the slope is noticeable in the entire range of variation of the inhomogeneity.

When $N = 10$ for both types of inhomogeneity, the same effects were observed as for $N = 3$. There was no change in the nature of the effect of inhomogeneity on $p_s(S)$, although the change in the number of layers itself, of course, affected the view of $p_s(S)$. The graphs for $N = 10$ were shifted towards smaller "window" areas with the same r relative to the graphs at $N = 3$. The graphs were built for $r = 0.2R...0.6R$; for less r at $N = 10$, few "windows" are formed (most numerical experiments produce opaque samples). An increase in the number of layers produces an effect similar to decreasing the radius r of islands — the graphs for $N = 10$, $r = 0.2R$ resemble graphs for $N = 3$, $r = 0,1R$, etc.

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