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Multifractal approach to assessing the heterogeneity of carbon alloys

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A multifractal structural analysis of carbon alloy structures after multi-stage flow processing was carried out. The statistical dimensions of the D-300 structure varied from 4.18 to 2.47, indicating the compactness of filling the space with martensite, bainite and pearlite. Indicators of the statistical dimension of cementite in the range from 2.18 to 1.55 characterize the dimension of the D300 structure. The fractal D0, information D1, correlation D2 dimensions of martensite, bainite, pearlite also varied in the range from 2.66 to 2.13, indicating the heterogeneity of the structure. A one-to-one correspondence between the multifractal indicators of the structure and the hardness of the iron-carbon alloy was established.
Keywords: carbon alloys, multifractal, structure, hardness, statistical dimensions, orderliness, regularity.

Мультифрактальний підхід до оцінки неоднорідності вуглецевих сплавів. *Д.Б. Глушкова, В.М. Волчук*

Проведено мультифрактальний аналіз структури вуглецевого сплаву після багатоступінчатої потокової обробки. Статистичні розмірності структури D-300 змінювалися від 4.18 до 2.47, що свідчить про компактність заповнення простору мартенситом, бейнітом, перлітом. Показники статистичної розмірності цементиту описують розмірність D300, що знаходяться в інтервалі від 2.18 до 1.55. Фрактальні D0, інформаційні D1, кореляційні D2 розмірності мартенситу, бейніту, перліту також змінювалися в діапазоні значень з 2.66 до 2.13, що свідчить про неоднорідність структури. Встановлена взаємо однозначна відповідність між мультифрактальними показниками структури та твердістю залізовуглецевого сплаву.

1. Introduction

Establishing the relationship between the structure and quality criteria of materials is one of the main tasks of materials science. However, the real structure of many materials has a complex morphology [1-3]. Such structures include the surfaces of materials after various types of spraying, heat treatment, non-metallic inclusions, etc. [4-7]. The structure of iron-carbon alloys is formed as a result of the decomposition of austenite and has a complex geometric configuration. For the quantitative identification of such structural elements as martensite, bainite, pearlite, carbides, and interphase boundaries, the language of fractal geometry [8-10], in particular multifractal theory, is successfully used [11].

Multifractal theory is a powerful tool for analyzing heterogeneous structures that exhibit complex hierarchical organization at different scales. In materials science, its application allows for a deeper understanding of the relationship between the microstructure of a material and its macroscopic properties.

Materials often have complex morphology due to random or deterministic processes of crystal grain growth, porosity, impurity distribution, etc. [12-15]. Multifractal analysis allows us:

– to assess the non-uniformity of phase distribution in composites, metals and ceramics;

– to identify patterns in the distribution of pores in porous materials;

– to describe the multiscale organization of grains in polycrystals.

The multifractal approach makes it possible to quantitatively assess the mechanical heterogeneity of a material, which directly affects its strength, plasticity and fracture resistance. For example, the analysis of the multifractal stress spectrum in materials allows us to predict their resistance to fatigue failure, as well as to identify patterns of local plastic deformation in metal alloys.

The multifractal approach is effective in studying the topography of materials, especially when analyzing surfaces subjected to wear, corrosion, or other degradation processes, as well as in studying the structure of surface defects in thin films, assessing wear and surface roughness of structural materials, and studying the dynamics of corrosion destruction of metals.

Multifractal theory provides a wide range of tools for the analysis of complex structures in materials science. It makes it possible not only to quantitatively assess the heterogeneity of materials, but also to establish regularities between their micro- and macrostructure. This opens up new opportunities for predicting mechanical, electrophysical and other operational characteristics of materials, which is important for the creation of new high-performance materials with specified properties [16].

The main goal of this study is to apply the fractal approach to analyze the heterogeneity of the surface structure of carbon alloys, as well as to establish the relationship between the spectrum of dimensions of structural elements and the mechanical characteristics of the material. To achieve this goal, the following tasks must be solved:

1. Study of interphase boundaries and morphological features of the alloy structure.

2. Assessment of the level of material heterogeneity using multifractal analysis.

3. Determination of correlations between multifractal characteristics of the structure and alloy hardness.

2. Experimental

Carbon alloy samples with the chemical composition presented in Table 1 were studied.

The microstructure of the samples was analyzed using a Neophot 2 optical microscope, and the resulting images were captured with an Olympus C-50 digital camera with a resolution of 2288×1712 pixels. This allowed us to obtain detailed images necessary for further processing and analysis.

The images were presented in BMP format with 256 grayscale levels, which provides sufficient color depth for analyzing the heterogeneity of the material structure. The use of the gray color spectrum allows for a more accurate assessment of interfacial boundaries, microdefects, and the distribution of structural components.

Figure 1 shows the microstructure of SSh-KhNM-55 cast iron after multi-stage heat treatment.

The obtained microstructure images will be used for further multifractal analysis, which will allow establishing quantitative characteristics of the material heterogeneity and the relationship between the dimensional spectrum and the mechanical properties of the alloy.

3. Results and discussion

Multifractal analysis is based on the assessment of statistical characteristics of elements of a metal structure, which are calculated on the basis of the Renyi spectrum of statistical dimensions $D(q)$. It allows us to determine the most significant contribution to the statistical sum $\sum_{i=1}^{N} p_i^q$ for given values of the exponent q. This statistical sum reflects the probability distribution over all points of the studied surface [17]:

Table 1. Chemical composition of carbon alloy

C	Yes	Mn	P	S	Cr	Ni	Mo	Cu	V	Mg
2.95	1.22	0.56	0.034	0.012	0.63	3.55	0.43	0.10	0.008	0.058

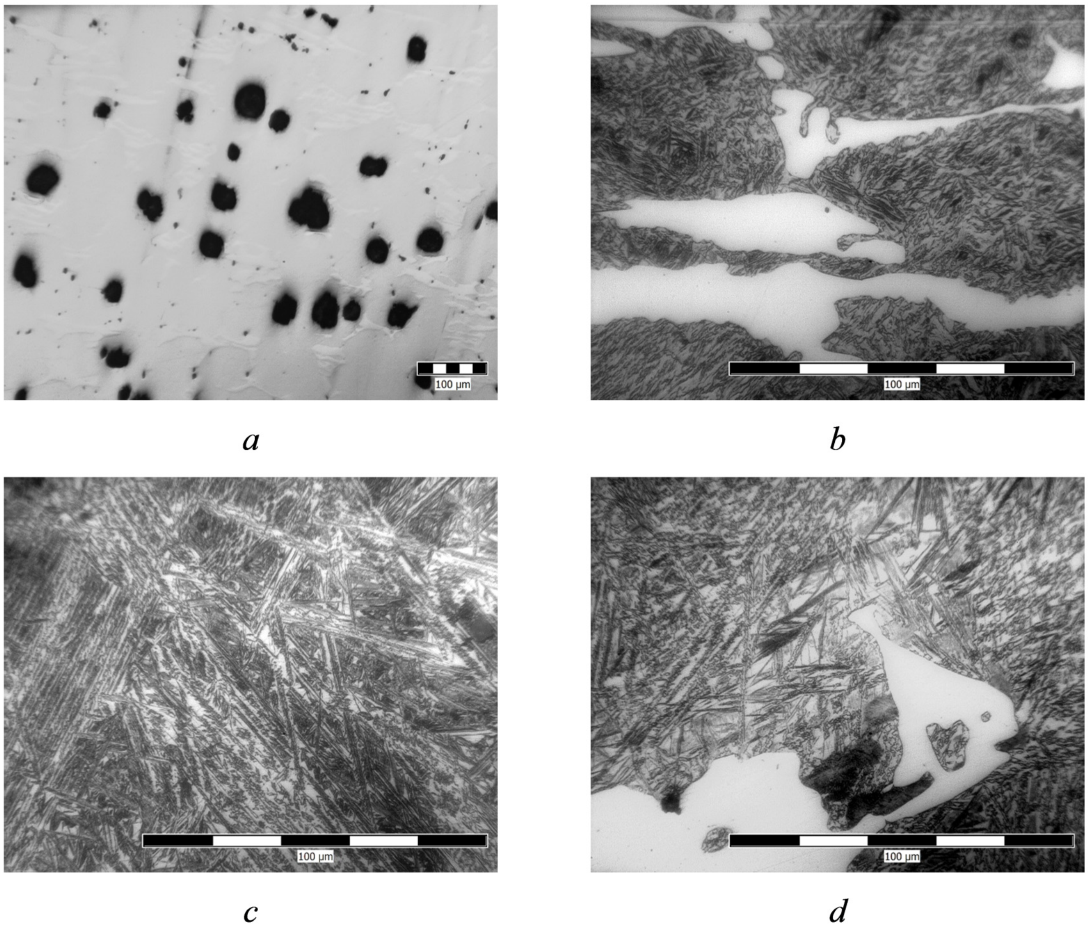

Fig. 1. Microstructure of carbon alloy: *a* - spheroidal graphite ~3%; *b* - matrix: Bainite + Martensite + Cementite (~18%); *c* - matrix: Martensite + Pearlite (~5%) + Cementite (~13%); *d* - matrix: Martensite + Bainite + Sorbite-like Pearlite + Cementite (~13).

$$D(q) = \frac{1}{q-1}\lim_{\varepsilon\to\infty}\frac{\ln\sum_{i=1}^{N} p_i^q}{\ln\varepsilon}, \qquad (1)$$

where p_i is the probability of detecting the studied point (computer pixel) belonging to the object in the i-th cell of a square grid of size ε. In this work, the value of the exponent q varied from –300 to 300.

To determine the degree of heterogeneity of the structure, the spectrum of singularities $f(\alpha)$ was calculated: This spectrum is described by the filling of square cells ε with equal probabilities $p_i(\varepsilon) \approx \varepsilon^{\alpha}$.

$$\begin{cases} \alpha = \dfrac{d\tau(q)}{dq}, \\ f(\alpha) = q\alpha - \tau(q) \end{cases} \qquad (2)$$

The spectrum *f(a)* was calculated by performing the Legendre transform of the function *τ(q)*.

Based on the results of the analysis of the spectrum of statistical dimensions $D(q)$ and the spectrum of singularities $f(\alpha)$, the statistical characteristics of the structure were calculated and the following results were obtained.

• The heterogeneity of the structure is determined by the uneven distribution of points over the regions into which the structure is divided, i.e. its geometrically identical elements are filled with points with different probabilities. In this case, the left or right part of the spectrum $f(\alpha)$ is different from zero. From the point of view of materials science, such heterogeneity determines the local defectivity of the studied structure, its porosity or roughness of individual elements;

• Orderliness $\Delta = D_1 - D_{300}$ and regularity $K = D_{-300} - D_{300}$. Dimensionality D_1 is called information dimension and is calculated from the spectrum of dimensions at q =1. These characteristics describe the degree of symmetry violation in the structure or the level of

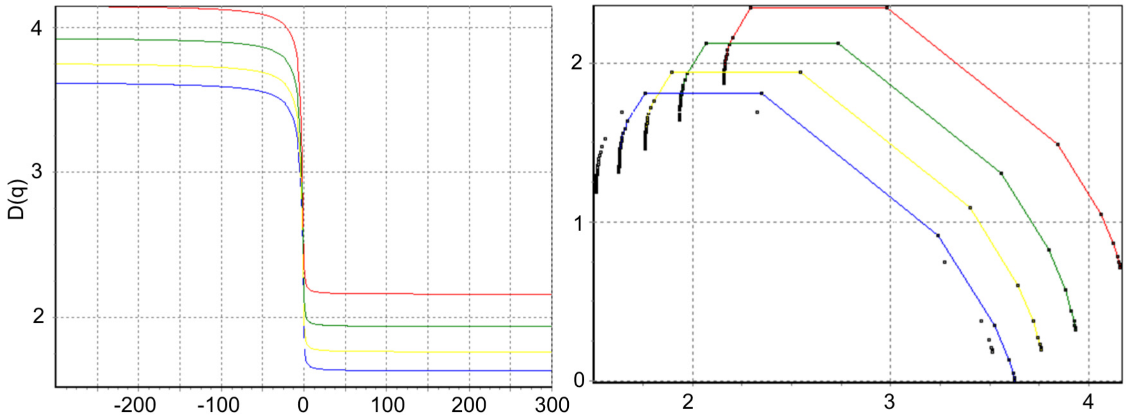

Fig. 2. Renyi spectrum of statistical dimensions $D(q)$ (*a*), and heterogeneity $f(\alpha)$ of the carbon alloy structure (*b*).

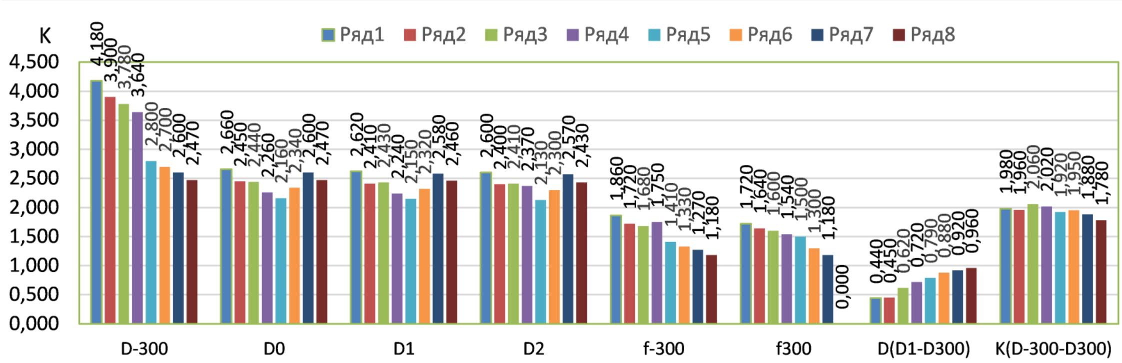

Fig. 3. Multifractal indicators of the heterogeneity of the structure of a carbon alloy.

non-equilibrium state of the system. The higher the numerical values of the indices Δ and K, the greater the content of periodic components (repeating structural elements of one phase) in the structure, and therefore, the more ordered it is.

Fig. 2 presents the results of calculations of the spectra of functions $D(q)$ and $f(\alpha)$ for the microstructure of the carbon alloy.

Statistical dimension D_{-300} changed from 4.18 to 2.47 (Fig. 3), which indicates the compactness of filling the space with martensite, bainite, pearlite. The statistical dimensionality of cementite is described by the dimension D_{300}, which is in the range from 2.18 to 1.55. Fractal D_0, information D_1, correlation D_2 dimensions of martensite, bainite, pearlite also changed in a wide range (Fig. 3).

Inhomogeneity indices of martensite, bainite, pearlite f_{-300} changed from 1.18 to 1.86, the indices of cementite heterogeneity f_{-300} varied from 0 to 1.72 (Fig. 3).

Multi-parameter model describing the influence of the indicators of heterogeneity and fractality of the structure on the hardness indicators:

$$HSD = 60.05 + 12.42 \cdot D_{600} + \\ + 3.429 \cdot D_{600} - 1.57 \cdot \Delta - 32.62 \cdot K - 2.14 \cdot D_0 + \\ + 0.85 \cdot D_1 - 3.45 \cdot D_0 \cdot R^2 = 0.89.$$

The obtained results indicate the possibility of identifying complex geometrically and heterogeneous structures of carbon alloys after multi-stage thermal treatment.

4. Conclusion

During the work, the following results were obtained:

1. The interphase boundaries and morphological features of the carbon alloy structure were investigated, indicating a fractal structure and a complex configuration of structural elements, which are difficult to describe using Euclidean geometry.

2. Assessment of the level of material heterogeneity using multifractal analysis was carried out.

3. A mathematical model has been constructed that describes the dependence of multifractal indicators of the structure and hardness of the alloy with the pair correlation coefficient $R^2 = 0.89$.

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