Self-accelerating Painleve-II soliton: a curious mathematical trick or fundamental physics?

A.D.Suprun

T.Shevchenko Kyiv National University, Department of Physics, 6 Glushkov Ave., 03022 Kyiv, Ukraine

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General dynamic properties of solitonic solutions are considered for one simplest special case of generalized nonlinear Schroedinger equation having the form $i\varphi_t + \varphi_{xx} - W(t,x,|\varphi|^2)\varphi = 0$ where $W(t,x,|\varphi|^2) = -|\varphi|^2 - xF(t)$. This case is of importance in numerous applications. The force $F(t)$ being absent and the general solitonic conditions being met, self-accelerating solutions are obtained referred to as Painleve-II solitons. Basing on these solutions, the strict statement of classical non-uniform dynamics for the traditional soliton localizat ion center of the equation under consideration has been shown to be possible.

В работе проанализированы общие динамические свойства солитонных решений одного из простейших, но важных в приложениях частных случаев обобщенного нелинейного уравнения Шредингера вида: $i\varphi_t + \varphi_{xx} - W(t,x,|\varphi|^2)\varphi = 0$, где $W(t,x,|\varphi|^2) = -|\varphi|^2 - xF(t)$. При отсутствии силы $F(t)$ и выполнении общих солитонных условий получены самоускоряющиеся решения. Они названы Painleve-II-солитонами. На базе этих решений, при $F(t) \neq 0$, показана возможность строгого построения классической неравномерной динамики для центра локализации традиционных солитонов рассматриваемого уравнения.

In this work, the "classical" solitonic solutions of nonlinear Schroedinger equation (NSE)

$$i\varphi_t + \varphi_{xx} + |\varphi|^2\varphi = 0, \quad (1)$$

will be studied from the standpoint of the possible realization of their non-uniform dynamics under conservation of the solitons themselves in their "classical" sense. The Eq.(1) can be applied in a wide variety of fields, including the functional properties of proteins [1–6], nonlinear optics [7–9], nonlinear acoustics [9–12], superconductivity models [13], theoretical models of physical vacuum[14, 15], and others [16]. An important advance in the examination of Eq.(1) is the inverse scattering method [17].

The solitonic solutions arising in this case possess an unique property referred to in physics as duality. This is manifested in that the wave function φ is a planet wave modulated in amplitude. The modulation character has a clearly pronounced particle-like form called soliton. This soliton moves in space uniformly, independent of the behavior of the wave solution part and without any damping. Since this solution is of basic importance for the problem in question, let it be considered in more detail. In the general case, the solution of Eq.(1) should be sought in the form

$$\varphi = U\exp(iV). \quad (2)$$

If, however, it is just the modulated plane wave that is the matter, the phase V takes a special form:

$$V = k \cdot x - \omega \cdot t, \quad (3)$$

where the constant k has the sense of the dimensionless wave number while the ω one,