

Kinetics of extreme values

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Effect of additive and multiplicative noise on linear systems has been considered. A closed kinetic equation has been derived for the distribution function of extreme values. Distinctions in the closing scheme as compared to derivation of the usual Focker-Planck type kinetic equation are discussed.

Рассмотрено влияние аддитивного и мультипликативного шума на линейные системы. Получено замкнутое кинетическое уравнение для функции распределения экстремальных значений. Обсуждены отличия в схеме замыкания по сравнению с выводом обычного кинетического уравнения типа Фоккера-Планка.

Evolution of most physical systems is of random character [1] and is described by various random type processes. Trajectories of continuous random processes attain local extremes (maxima and minima) at certain time points [2]. These extremes can result in failure of the whole system or a certain part thereof. Therefore, statistical characteristics of the extremes are to be known to evaluate the operating reliability of such systems. Those characteristics include distributions of extreme times and values as well as their number per a finite time interval.

The description problem of the random curve extreme distribution was first formulated and solved by S.O.Rice [3]. Later, the trajectory extremes of stationary random processes were studied in numerous theoretical and experimental works [2, 4–8]. The methods developed in those works are unsuitable to describe the extremes of non-stationary random processes of various classes.

In this work, the attention is given mainly to study of the distribution function of extreme values for systems subjected to external stochastic factors. Such systems are described by stochastic differential equations containing an external random process. Two simple types of equations with additive and multiplicative noise are considered. Statistical properties of the external random process are assumed to be known, it is assumed to be a stationary and Gaussian one. Statistical properties of the system are of course non-stationary, and in the case of multiplicative noise, those are also non-Gaussian. Kinetic equations for the extreme value distribution function have been obtained. Principal distinctions associated with closing of the kinetic equations are discussed. The close kinetic equation has been demonstrated to arise when additional equalities existing in such systems are used.

1. Extreme value distribution function

Let us discuss the probabilistic description of extremes for a random process $x(t)$ that is continuous and double-differentiable in the root mean square sense [2].